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Performance Comparison between OQAM and *N*-continuous OFDM

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ABSTRACT

Orthogonal frequency division multiplex/offset QAM (OFDM/OQAM) and *N*-continuous OFDM are both improved multi-carrier modulation (MCM) techniques for the transmission of signals over multipath fading channels. In this paper, we aim to compare these two techniques with respect to out-of-band radiation, bit error ratio (BER) and calculation complexity. Analysis and simulation results show that, compared to the *N*-continuous OFDM, the OFDM/OQAM has lower out-of-band radiation, calculation consumption and similar BER performance.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is an efficient scheme and it has been some practical applications. However, the traditional OFDM has some intrinsic drawbacks that the discontinuous phase between adjacent symbols leads to high out-of-band radiations and its robustness to multi-path propagation effect is acquired by the insertion of a cyclic prefix (CP). To alleviate these drawbacks, another MCM schemes such as N-continuous OFDM and OFDM/offset QAM (OFDM/OQAM) are proposed in [1,2] respectively. Compared to traditional CP-OFDM, OFDM/OQAM may provide a higher useful bit rate since it operates without CP and N-continuous OFDM has less out-of-band radiation since it has several continuous order of derivatives at the edge of adjacent OFDM symbols.

For OFDM/OQAM system, pulse shapes with good time-frequency localization (TFL) can be introduced. Among these, a pulse shape named extended Gaussian functions (EGF) is widely used and its TFL can be adjusted by the time and frequency real parameters [3]. Through selecting a suitable parameter, an OFDM/OQAM signal with very low out-of-band radiation can be acquired [4]. On the other hand, in the traditional OFDM system, each OFDM symbol is independently and the transmitted signals are not continuous, leading to high out-of-band radiation. In order to overcome the discontinuities between consecutive OFDM symbols, N-continuous OFDM scheme is proposed at the price of increasing calculation complexity and/or system per-

formance. Therefore, we have to comprise between these aspects for practical application.

In this paper, we compare the performance of OFDM/OQAM and N-continuous OFDM. Firstly, the system models of them are given in section II. And then we analyze the out-of-band radiation, bit error ratio (BER) and calculation complexity of these two schemes in section III. The simulation results are shown in section IV. Brief conclusion is given in section V. The analysis and simulation results show that, compared to the N-continuous OFDM, the OFDM/OQAM system has similar system performance, lower out-of-band radiation and less calculation consumption.

2. System Model

2.1. System Model of OFDM/OQAM System

The baseband version of a continuous-time OFDM/OQAM transmitting signal can be written as [2]

$$s(t) = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{M-1} a_{m,n} e^{j\phi_{m,n}} e^{j2\pi m\upsilon_0 t} g(t - n\tau_0), \qquad (1)$$

with M an even number of sub-carriers, $a_{m,n}$ the real-valued symbol conveyed by the sub-carrier of index m during the symbol time of index n υ_0 the subcarrier spacing and τ_0 the time offset between the adjacent real part and imaginary part of an OFDM/OQAM symbol. $\upsilon_0 = 1/T_0 = 1/2\tau_0$, with T_0 the duration of the complex-valued symbols. $\phi_{m,n}$ is an additional phase term given by

$$\phi_{m,n} = \phi_0 + \frac{\pi}{2}(m+n) \quad (\text{mod} \ \pi), \quad (2)$$

where ϕ_0 can be arbitrarily chosen. $g(t)$ is the pulse shape that can be different from the rectangle shape of conditional OFDM system. The EGF shape is often used in OFDM/OQAM system and its out-of-band characteristic can be adjusted by the Gaussian parameter α. In the following, the adaption and compromise between BER performance and out-of-band radiation are discussed.

For a distortion-free channel, perfect reconstruction of real symbols is obtained owing to the following real orthogonal condition

$$\Re\left\{\left\langle g_{m,n} \mid g_{p,q} \right\rangle\right\} = \Re\left\{\int g_{m,n}(t) g_{p,q}^*(t)\right\} = \delta_{m,p}\delta_{n,q} \quad (3)$$

where $\Re\{\bullet\}$ is the taking real part operator. $\delta_{m,p} = 1$ if $m = p$ and $\delta_{m,p} = 0$ if $m \neq p$.

2.2. System model of N-continuous OFDM System

The baseband equivalent OFDM symbol can be given as

$$s(t) = \sum_{i=-0}^{+\infty} s_i\left(t - i\left(T_s + T_g\right)\right) \quad (4)$$

where T_s is the OFDM symbol duration and T_g is the CP length. In order to make the transmitted signal $s(t)$ and its first N derivatives continuous, the following equation should be satisfied that

$$\frac{d^n}{dt^n} s_i(t)\bigg|_{t=-T_g} = \frac{d^n}{dt^n} s_{i-1}(t)\bigg|_{t=T_g}, \quad (5)$$

for all $i \geq 1$ and n=0, 1,…,N [1,5].

The realization of N-continuous OFDM symbol is to pre-coder the set of information symbols $\{d_{k,i}\}$ (taken from some complex-valued symbol constellation $\mathbb{C}$) in each OFDM symbol such that $s(t)$ becomes N-continuous. Concretely, the OFDM symbol $s_i(t)$ can be expressed as [1]

$$s_i(t) = \sum_{k \in \Omega} \overline{d}_{k,i} e^{j2\pi \frac{k}{T_s}t} \quad -T_g \leq t \leq T_s, \quad (6)$$

where $\Omega = \{k_0, k_1, ..., k_{K-1}\}$ and the complex numbers $\overline{d}_{k,i}$ are the result of pre-coding information symbols $d_{k,i}$. For OFDM symbols, the characterization (5) becomes

$$\sum_{k \in \Omega} k^n e^{j\phi k} \overline{d}_{k,i} = \sum_{k \in \Omega} k^n \overline{d}_{k,i-1}, \quad (7)$$

where $\phi = -2\pi T_g / T_s$. An equivalent form of vector is that

$$\mathbf{A}\Phi\overline{\mathbf{d}}_{\mathbf{i}} = \mathbf{A}\overline{\mathbf{d}}_{\mathbf{i-1}}, \quad (8)$$

where

$$\overline{\mathbf{d}}_{\mathbf{i}} = \left(\overline{d}_{k_0,i}, \overline{d}_{k_1,i}, ..., \overline{d}_{k_{K-1},i}\right)^T, \quad (9)$$

$$\Phi = diag\left(e^{j\phi k_0}, e^{j\phi k_1}, ..., e^{j\phi k_{K-1}}\right), \quad (10)$$

and

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & ... & 1 \\ k_0 & k_1 & ... & k_{K-1} \\ ... & ... & & ... \\ k_0^N & k_1^N & ... & k_{K-1}^N \end{pmatrix}. \quad (11)$$

Finally, we can get $\overline{\mathbf{d}}_{\mathbf{i}} = (\mathbf{I} - \mathbf{P})\mathbf{d}_i + \mathbf{P}\Phi^{\mathbf{H}}\overline{\mathbf{d}}_{\mathbf{i-1}}$, where $\mathbf{P} = \Phi^H \mathbf{A}^H \left(\mathbf{A}\mathbf{A}^{\mathbf{H}}\right)^{-1} \mathbf{A}\Phi$. The deducing detail can be referred to [1].

3. Comparison of the OFDM/OQAM and N-continuous OFDM System

3.1. Comparison of the Out-of-band Radiation

Because the pulse shape with lower side lobe such as the EGF shape is used in the OFDM/OQAM system, the final transmitting signal has very low out-of-band radiation naturally. The EGF shape is defined as [6]

$$z_{a,\upsilon_0,\tau_0}(t) = \frac{1}{2}\left[\sum_{k=0}^{\infty} d_{k,a,\upsilon_0}\left[g_\alpha\left(t + \frac{k}{\upsilon_0}\right) + g_\alpha\left(t - \frac{k}{\upsilon_0}\right)\right]\right] \cdot \sum_{l=0}^{\infty} d_{l,1/\alpha,\tau_0} \cos\left(2\pi l \frac{t}{\tau_0}\right) \quad (12)$$

where $g_\alpha(t)$ is the Gaussian function defined by

$$g_\alpha(t) = (2\alpha)^{1/4} e^{-\pi\alpha t^2} \quad a > 0.$$

υ_0 is the frequency parameter, τ_0 is the time parameter and $\upsilon_0\tau_0 = 1/2$. For different a, we can get different transmitting signal and the larger the value of a is, the lower out-of-band radiation will be.

For N-continuous OFDM system, the out-of-band radiation is determined by the number of orders N that having continuous derivatives. And the larger N is, the lower out-of-band radiation will be.

In contrast to OFDM/OQAM, increasing of N leads to not only the decrease of BER performance but also the rapid increasing of calculation complexity. Therefore, the compromise should be made between calculation consumption and system performance.

3.2. Comparison of the BER Performance

In OFDM/OQAM system, since the pulse shape that having immunity to the inter-symbol interference (ISI) is introduced, the CP is not necessary. Then it can provide higher bit rate or equivalently higher bit energy.

While for the N-continuous OFDM, the using of CP cannot be avoided. Furthermore, since the maximum-likelihood detector of N-continuous is prohibitively

complex, suboptimal iterative detector is usually used and there is a certain performance loss when the iterative time is less than 4.

3.3. Comparison of the Calculation Complexity

In the OFDM/OQAM system, the complex data sources are divided into real and image part and carried out FFT calculation respectively. Then a pulse shape with the duration of $T_c = mT_0$ is met. Finally these two paths signals are added and we get the transmitting OFDM/OQAM signal [4]. Therefore the OFDM/OQAM system calculation consumption comprises of

$$2M \log_2 M + 4mM$$

real multiplications and $2M \log_2 M + 4(m-1)M$ real additions.

For N-continuous OFDM, the system calculation consumption comprises of $8M^2$ real multiplications and $8M^2 - 2M$ real additions. Because both multiplications and additions calculation are directional proportional to the square of the number of subcarriers M. Therefore the calculation consumption increases rapidly for a large M.

Taking the number of subcarriers $M = 256$ and the length the of pulse shape is four times of the length of OFDM symbol, i.e., $m = 4$, for example, the numerical comparison of calculation complexity is listed in **Table 1**. It is shown that the calculation complexity of N-continuous OFDM is far more than that of the OFDM/OQAM system.

4. Simulation Results

In this section, it aims to compare the out-of-band radiation and BER performance of OFDM/OQAM with the N-continuous system. Unless other wise stated, the simulation parameters are: the number of subcarriers is M = 512, sampling time Ts = 1/15 ms, CP length Tg = 144Tsamp, the data is modulated by 16QAM. A 3GPP EVA fading channel is combined without carrier frequency offset and synchronization errors. The power spectrum is estimated by Welch's averaged periodogram method with sampling duration Tsamp, a 4096-sample Hanning window and 512-sampleoverlap. And the simulation results are shown in **Figure 1** and **Figure 2**.

Figure 1 shows the results of BER performance of N-continuous OFDM and OFDM/OQAM, and the results of original OFDM are also given. It can be seen that N-continuous OFDM has poorer performance than original

Table 1. Numerical comparison of calculation complexity.

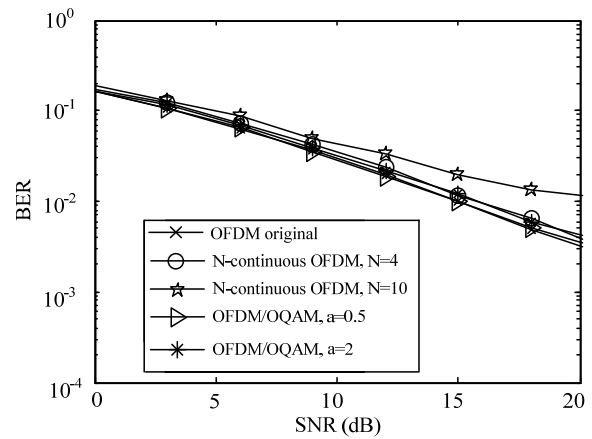| scheme	real multiplication	addition real
OFDM/OQAM	8192	7168
N-continuou OFDM	524288	523776

Figure 1. Comparison of BER performance of N-continuous OFDM and OFDM/OQAM.
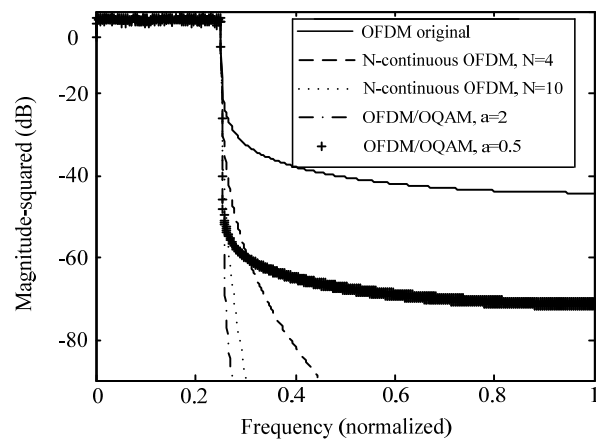

Figure 2. Comparison of out-of-band radiation of N-continuous OFDM and OFDM/OQAM.

OFDM and the situation is more serious when the N is up to 10. On the other hand, for OFDM/OQAM system, it has similar performance with origin OFDM, and the performance loss is negligible even for $\alpha = 2$.

In **Figure 2**, it shows the power spectral density of N-continuous OFDM and OFDM/OQAM, together with the results of original OFDM. The results imply that the N-continuous OFDM has lower out-of-band radiation than original OFDM. And the larger N is, the lower out-of-band radiation will be. While for OFDM/OQAM system, it has a sharp decay at the edge of the main lobe, which is better for transmission in means of avoiding adjacent interference. And the parameter α has a certain effect on the out-of-band radiation. When N = 10, N-continuous OFDM is a little better than that of OFDM/OQAM with $\alpha = 0.5$. But at same time, the calculation consumption is very large.

5. Conclusions

In this paper, the performance of OFDM/OQAM and

N-continuous OFDM are compared with respect to out-of-band radiation, BER performance and calculation complexity with different system parameters. We show that N-continuous OFDM is only designed to suppress the out-of-band radiation, but this benefit is acquired with large calculation consumption increase. The advantages of OFDM/OQAM lie in that it has not only the nature advantage of low out-of-band radiation, but also the strong immunity to ISI and inter-carrier interference simultaneously. When the length of pulse shape is suitable selected, while with moderate complexity increase. In whole, OFDM/OQAM system outperforms the N-continuous OFDM system.

6. Acknowledgements

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