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Analysis of the Impact of Optimal Solutions to the Transportation Problems for Variations in Cost Using Two Reliable Approaches

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Abstract

In this paper, we have used two reliable approaches (theorems) to find the optimal solutions to transportation problems, using variations in costs. In real-life scenarios, transportation costs can fluctuate due to different factors. Finding optimal solutions to the transportation problem in the context of variations in cost is vital for ensuring cost efficiency, resource allocation, customer satisfaction, competitive advantage, environmental responsibility, risk mitigation, and operational fortitude in practical situations. This paper opens up new directions for the solution of transportation problems by introducing two key theorems. By using these theorems, we can develop an algorithm for identifying the optimal solution attributes and permitting accurate quantification of changes in overall transportation costs through the addition or subtraction of constants to specific rows or columns, as well as multiplication by constants inside the cost matrix. It is anticipated that the two reliable techniques presented in this study will provide theoretical insights and practical solutions to enhance the efficiency and cost-effectiveness of transportation systems. Finally, numerical illustrations are presented to verify the proposed approaches.

Keywords

Transportation Problem, Initial Basic Feasible Solution, Optimal Solution, Two Reliable Approaches (theorems) and Numerical Illustrations

1. Introduction

One of the first uses of linear programming is the transportation problem (TP). It handles the movement of products between many supply points (sources) and several demand locations (destinations). It is possible to construct a wide range of decision issues as TPs, including production distribution, job scheduling, inventory control, and investment analysis. Determining the delivery plan that minimizes overall shipping costs while meeting supply and demand restrictions is the primary goal of the transportation challenge. In operations management, the application of transportation problems for variations in cost is crucial for optimizing supply chain logistics and distribution networks. Let's consider a manufacturing company that sources raw materials from multiple suppliers and distributes finished products to diverse markets. Fluctuations in fuel prices, labor costs, or other variables can significantly impact transportation expenses. By employing transportation optimization models, operations managers can design efficient delivery routes, select appropriate carriers, and allocate resources effectively. In the face of cost variations, the system can dynamically adjust to minimize transportation expenses. For instance, if fuel costs rise unexpectedly, the optimization model might recommend consolidating shipments, re-routing deliveries, or even exploring alternative transportation modes to maintain cost efficiency. This application of transportation problems in operations management ensures that the supply chain remains agile and responsive to changing cost dynamics. It not only helps control operational expenses but also contributes to overall efficiency and customer satisfaction by ensuring timely and cost-effective deliveries. This approach is particularly relevant in industries where transportation costs form a significant portion of the overall operational expenses, such as manufacturing, retail, and distribution. The foundational transportation model, initially proposed by Hitchcock [1], underwent advancements by Koppmans [2] and Danzig [3]. Several heuristic techniques, including the North-West corner rule, least cost approach, Vogel's approximation method (VAM) [4], Russell's approximation method [5], and other renowned heuristics, were employed to derive a simple and practical solution. Kirca and Satir [6] introduced the total opportunity cost approach (TOM) as a heuristic for obtaining the initial viable solution, later expanded by Mathirajan and Meenakshi [7] using VAM. Korukoglu and Balli [8] proposed an enhanced VAM based on total opportunity cost (TOC). Charnes and Cooper [9] devised the stepping stone method, while Dantzig [10] introduced the modified distribution (MODI) approach for verifying optimality in the initial fundamental solution. Numerous scholars, including Amaliah *et al.* [11], Hosseini [12], Jude *et al.* [13], Juman and Hoque [14], Karagul and Sahin [15], and Uddin and Khan [16], have delved into the exploration of Initial Basic Feasible Solution (IBFS) as a means to directly attain the optimal solution for Transportation Problems (TP) without relying on Stepping Stone or Modified Distribution (MODI) methods. Rashid [17] contributed a theorem in the context of resolving transportation issues. Ahmed *et al.* [18] introduced an incessant allocation method to analyze and minimize transportation cost. Taha [19], studied for finding optimal solution of transportation problems. Amaliah *et al.* [20], introduced a heuristic method to find the initial basic feasible solution of transportation problem (TP).

In the existing literature, the authors obtain that the optimal solution of Transportation Problems (TP) without incorporating changes to the Transportation cost. In general, various communication systems in real-world applications demand representation through variations in Transportation cost, particularly for multiservice facilities catering to customers. This investigation addressed in this paper by integrating the variation of Transportation cost. The presented models emerge as a more reliable framework when compared to the existing models in the literature. An efficient transportation plan not only reduces costs but also improves overall operational efficiency. This can lead to reduced lead times, faster order fulfillment, and enhanced customer satisfaction. The main objective of this paper is to examine the impact of fluctuations in transportation costs on reliability. Significant changes in costs can disrupt the optimal transportation plan and impact various aspects of an organization's operations. Adapting to these changes effectively through cost management strategies, contingency planning, and informed decision-making is crucial for maintaining efficiency, competitiveness, and financial stability in dynamic business environments. These two suggested theorems have the following benefits:

1) Analyzing the effect on transportation expenses of possible disruptions or modifications in the availability of resources.

2) The two suggested theorems in the business sector can be used to examine how sensitive transportation costs are to modifications in the component parts of the cost matrix.

3) Quickly modifying transportation expenses without sacrificing the best possible outcome in order to effectively manage resources.

4) Compared to starting from scratch and solving the problem, finding the best solution for the modified problem typically requires less computing work.

This paper is organized as follows: Section 2: mathematical formulations; Section 3: description of the models; Section 4: analysis of the proposed theorems; Section 5: numerical examples; Section 6: discussion of results; and the last section: the conclusion of the paper.

2. Mathematical Formulation

Let m suppliers (sources) and n customers (destinations) be considered. The m suppliers can ship a single homogeneous product to any of the n customers at a shipping cost per unit c_{ij}. Let a_i represent the number of supply units needed at source $i\left(i=1,2,\cdots,m\right)$, b_j represent the number of demand units needed at destination $j\left(j=1,2,\cdots,n\right)$ and x_{ij} represent the unknown quantity that needs to be transported from source i to destination j. Consequently, the corresponding linear programming model will be

Minimize (total cost) $z=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad \text{for } i = 1, 2, \cdots, m \ (\text{supply constraint})$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad \text{for } j = 1, 2, \cdots, n \ (\text{demand constraint})$$

$$x_{ij} \geq 0.$$

In matrix form the model can be formulated as Table 1.

3. Description of the Model

In this section, we have shown two modified cost matrix using the addition or subtraction of constants $\alpha_i; i = 1, 2, \cdots, m$ and $\beta_j; j = 1, 2, \cdots, n$ to specific rows or columns within the matrix, along with the multiplication of constants k within the same cost matrix. Through these systematic modifications, the impact of changes to individual elements on the overall transportation cost structure can be precisely measured (Table 2 and Table 3).

The total transportation cost of modified cost matrix for TP is minimized by the following formula for Table 2.

$$z^* = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(c_{ij} \pm \alpha_i \pm \beta_j \right) x_{ij}$$

Table 1. Matrix form model.

	D_1	D_2	$\cdots$	D_n	Supply
S_1	x_{11} / c_{11}	x_{12} / c_{12}	$\cdots$	x_{1n} / c_{1n}	a_1
S_2	x_{21} / c_{21}	x_{22} / c_{22}	$\cdots$	x_{2n} / c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
S_m	x_{m1} / c_{m1}	x_{m2} / c_{m2}	$\cdots$	x_{mn} / c_{mn}	a_m
Demand	b_1	b_2	$\cdots$	b_n	

Table 2. Modified cost matrix.

	D_1	D_2	$\cdots$	D_n	Supply
S_1	x_{11} / $c_{11} \pm \alpha_1 \pm \beta_1$	x_{12} / $c_{12} \pm \alpha_1 \pm \beta_2$	$\cdots$	x_{1n} / $c_{1n} \pm \alpha_1 \pm \beta_n$	a_1
S_2	x_{21} / $c_{21} \pm \alpha_2 \pm \beta_1$	x_{22} / $c_{22} \pm \alpha_2 \pm \beta_2$	$\cdots$	x_{2n} / $c_{2n} \pm \alpha_2 \pm \beta_n$	a_2
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
S_m	x_{m1} / $c_{m1} \pm \alpha_m \pm \beta_1$	x_{m2} / $c_{m2} \pm \alpha_m \pm \beta_2$	$\cdots$	x_{mn} / $c_{mn} \pm \alpha_m \pm \beta_n$	a_m
Demand	b_1	b_2	$\cdots$	b_n	

Table 3. Modified cost matrix.

	D_1	D_2	$\cdots$	D_n	Supply
S_1	x_{11} / kc_{11}	x_{12} / kc_{12}	$\cdots$	x_{1n} / kc_{1n}	a_1
S_2	x_{21} / kc_{21}	x_{22} / kc_{22}	$\cdots$	x_{2n} / kc_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
S_m	x_{m1} / kc_{m1}	x_{m2} / kc_{m2}	$\cdots$	x_{mn} / kc_{mn}	a_m
Demand	b_1	b_2	$\cdots$	b_n	

Therefore, the total cost (objective function) becomes for Table 3.

$$z^* = k\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

4. Analysis of the Proposed Approaches (Theorems)

Different models have different assumptions and approaches. Using two models provides a more robust analysis that accounts for various scenarios and conditions. It helps identify the potential impact of model-specific assumptions on the results. These models help in understanding how variations in transportation costs affect the optimal solution under different modeling frameworks. In this section we have proposed two theorems to find the effect of optimal solution for transportation problem using variation of costs.

Theorem 4.1: The optimality of the solution to the original problem is maintained for the new problem if there is an addition or subtraction of a constant quantity α_i from each element of the *i*th row and/or a constant quantity β_j from each element of the *j*th column of the transportation cost matrix $\left[c_{ij}\right]$. Additionally, the total transportation cost is altered by

$$\sum_{i=1}^{m}(\pm\alpha_i)a_i + \sum_{j=1}^{n}(\pm\beta_j)b_j.$$

Proof: Let $\{x_{ij} : i = 1,2,\cdots,m;\ j = 1,2,\cdots,n\}$ represent the optimal solution in relation to the original cost matrix $\left[c_{ij}\right]$. Then, the objective function for total cost is provided by

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

If the real constants α_i and β_j are added (or subtracted) to (or from) *i*th row and *j*th column respectively of the cost matrix $\left[c_{ij}\right]$ and z^* indicates the entire cost of the modified cost matrix, $\left[c_{ij} \pm \alpha_i \pm \beta_j\right]$ after that

$$z^* = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij} \pm \alpha_i \pm \beta_j\right)x_{ij}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m}(\pm\alpha_i)\sum_{j=1}^{n} x_{ij} + \sum_{j=1}^{n}(\pm\beta_j)\sum_{i=1}^{m} x_{ij}$$

$$= z + \sum_{i=1}^{m}(\pm\alpha_i)a_i + \sum_{j=1}^{n}(\pm\beta_j)b_j$$

Since the terms that are added to z to give z^* are independent of x_{ij}, it follows that z^* is minimized whenever z is minimized or vice versa and the total transportation cost is changed by $\sum_{i=1}^{m}(\pm\alpha_i)a_i + \sum_{j=1}^{n}(\pm\beta_j)b_j$.

Theorem 4.2: If a constant amount is multiplied by each element in a transportation cost matrix $[c_{ij}]$, then the overall transportation cost is increased by the constant quantity times, and the best solution of the original problem remains optimal for the new problem.

Proof: Let $\{x_{ij} : i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\}$ represent the optimal solution in relation to the original cost matrix $[c_{ij}]$. Then, the objective function for total cost is provided by

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

If each element of the cost matrix $[c_{ij}]$ is multiplied by a constant quantity k then the total cost (objective function) becomes

$$z^* = \sum_{i=1}^{m}\sum_{j=1}^{n}(kc_{ij})x_{ij}$$
$$= k\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$
$$= kz$$

Hence if x_{ij} is the optimal solution of the cost matrix $[c_{ij}]$ then it is also optimal for the cost matrix $[kc_{ij}]$ and the total cost is increased by k times.

5. Numerical Illustrations

In real-life scenarios, the transportation problem can be found in various contexts such as supply chain management, logistics, and distribution planning, where minimizing transportation costs while meeting supply and demand requirements is a critical objective. These theorems provide practical methods to address these optimization challenges. Subsequently, we provide numerical examples to validate the introduced theorems.

Example 5.1: We investigate the following transportation problem to demonstrate Theorem 1.

	D_1	D_2	D_3	D_4	Supply
S_1	40	17	34	27	150
S_2	15	37	30	25	260
S_3	33	28	42	45	520
Demand	140	400	250	140	

To find the optimal cost of the TP we have used least cost method and then employing MODI method. The optimal transportation schedule is presented as Table 4.

Consequently, the optimal solution and the aggregate transportation cost are

$$x_{12} = 130,\ x_{14} = 20,\ x_{21} = 140,\ x_{24} = 120,\ x_{32} = 270,\ x_{33} = 250$$
$$z = 25910.$$

We observe the following, when we modified the cost matrix of original transportation problem:

If we add $\alpha_1 = 14$, $\alpha_2 = 2$, $\alpha_3 = 10$, $\beta_1 = 4$, $\beta_3 = 9$ and subtract $\beta_2 = 12$, $\beta_4 = 14$ to each unit cost in the original transportation problem by $\left[c_{ij} \pm \alpha_i \pm \beta_j\right]$, the modified cost matrix is given below:

	D_1	D_2	D_3	D_4	Supply
S_1	58	19	57	27	150
S_2	21	27	41	13	260
S_3	47	26	61	41	520
Demand	140	400	250	140	

To find the optimal cost of the modified TP we have also used least cost method and then applying MODI method. The optimal transportation schedule is as Table 5.

Table 4. Optimal solution original TP.

	D_1	D_2	D_3	D_4	Supply
S_1	40	130 17	34	20 27	150
S_2	140 15	37	30	120 25	260
S_3	33	270 28	250 42	45	520
Demand	140	400	250	140	

Table 5. Optimal solution of modified cost of TP.

	D_1	D_2	D_3	D_4	Supply
S_1	58	130 19	57	20 27	150
S_2	140 21	27	41	120 13	260
S_3	47	270 26	250 61	41	520
Demand	140	400	250	140	

Consequently, the optimal solution and the aggregate transportation cost are

$$x_{12} = 130,\ x_{14} = 20,\ x_{21} = 140,\ x_{24} = 120,\ x_{32} = 270,\ x_{33} = 250$$
$$z^* = 29780.$$

Thus we see that after adding or subtracting constant quantities α_i and β_j, the optimal schedule remains unchanged and the total transportation cost is changed by

$$\begin{aligned} d &= \sum_{i=1}^{3}(\pm\alpha_i)a_i + \sum_{j=1}^{4}(\pm\beta_j)b_j \\ &= \alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3 + \beta_1 b_1 - \beta_2 b_2 + \beta_3 b_3 - \beta_4 b_4 \\ &= 14\times150 + 2\times260 + 10\times520 + 4\times140 - 12\times400 + 9\times250 - 14\times140 \\ &= 3870 \end{aligned}$$

$$i.e.,\ d = z^* - z = 29780 - 25910 = 3870$$

Table 6 shows some random variations in costs and impact on optimal solution and total cost. The results are investigated through TORA optimization software.

Example 5.2: We investigate the following transportation problem to demonstrate Theorem 2.

	D_1	D_2	D_3	Supply
S_1	5	7	6	250
S_2	8	12	13	100
S_3	20	17	9	160
Demand	130	170	210	

Applying Vogel's approximation method for initial solution and then implementing MODI method the optimal transportation schedule is as Table 7.

Consequently, the optimal solution and the aggregate transportation cost are

$$x_{11} = 30,\ x_{12} = 170,\ x_{13} = 50,\ x_{21} = 100,\ x_{33} = 160$$
$$z = 3880.$$

If we multiply each element of $[c_{ij}]$ by a constant quantity $k = 5$ the modified cost matrix $[kc_{ij}]$ is

	D_1	D_2	D_3	Supply
S_1	25	35	30	250
S_2	40	60	65	100
S_3	100	85	45	160
Demand	130	170	210	

Solving the above problem by using Vogel's approximation method for initial solution and then employing the MODI method, the optimal transportation table is shown in Table 8.

Table 6. Optimal results in variations of cost of TP.

Values of parameters	Optimal solution	Optimal cost	Amount of Changing cost $d = z^* - z$
$\alpha_1 = 15,\ \alpha_2 = 20,\ \alpha_3 = -5,\ \beta_1 = -10,$ $\beta_2 = -12,\ \beta_3 = 35,\ \beta_4 = 18$	$x_{12} = 130,\ x_{14} = 20,\ x_{21} = 140,$ $x_{24} = 120,\ x_{32} = 270,\ x_{33} = 250$	$z^* = 35830$	9920 unit
$\alpha_1 = -25,\ \alpha_2 = -15,\ \alpha_3 = -18,\ \beta_1 = 14,$ $\beta_2 = 40,\ \beta_3 = 32,\ \beta_4 = 35$	$x_{12} = 130,\ x_{14} = 20,\ x_{21} = 140,$ $x_{24} = 120,\ x_{32} = 270,\ x_{33} = 250$	$z^* = 39760$	13850 unit
$\alpha_1 = 25,\ \alpha_2 = 15,\ \alpha_3 = 18,\ \beta_1 = 14,$ $\beta_2 = 40,\ \beta_3 = 32,\ \beta_4 = 35$	$x_{12} = 130,\ x_{14} = 20,\ x_{21} = 140,$ $x_{24} = 120,\ x_{32} = 270,\ x_{33} = 250$	$z^* = 73780$	47870 unit
$\alpha_1 = 25,\ \alpha_2 = 15,\ \alpha_3 = 18,\ \beta_1 = -5,$ $\beta_2 = -10,\ \beta_3 = -8,\ \beta_4 = -13$	$x_{12} = 130,\ x_{14} = 20,\ x_{21} = 140,$ $x_{24} = 120,\ x_{32} = 270,\ x_{33} = 250$	$z^* = 34400$	8490 unit

Table 7. Optimal solution of original TP.

	D_1	D_2	D_3	Supply
S_1	30 5	170 7	50 6	250
S_2	100 8	12	13	100
S_3	20	17	160 9	160
Demand	130	170	210	

Table 8. Optimal solution of modified cost of TP.

	D_1	D_2	D_3	Supply
S_1	30 25	170 35	50 30	250
S_2	100 40	60	65	100
S_3	100	85	160 45	160
Demand	130	170	210	

Consequently, the optimal solution and the aggregate transportation cost are

$$x_{11} = 30,\ x_{12} = 170,\ x_{13} = 50,\ x_{21} = 100,\ x_{33} = 160$$
$$z^* = 19400.$$

Thus it is observed that if we multiply each element of $\left[c_{ij}\right]$ by 5, the optimal schedule remains unchanged and the total transportation cost is increased by 5 times *i.e.* $z^* = kz = 5 \times 3880 = 19400$.

Following **Table 9** shows some random variations in costs and impact on

Table 9. Optimal results in variations of cost of TP.

Values of parameters	Optimal solution	Optimal cost	Amount of changing cost $d = z^* - z$
$k = 10$	$x_{11} = 30,\ x_{12} = 170,\ x_{13} = 50,\ x_{21} = 100,\ x_{33} = 160$	$z^* = 38800$	34,920 unit
$k = 15$	$x_{11} = 30,\ x_{12} = 170,\ x_{13} = 50,\ x_{21} = 100,\ x_{33} = 160$	$z^* = 58200$	54,320 unit
$k = 20$	$x_{11} = 30,\ x_{12} = 170,\ x_{13} = 50,\ x_{21} = 100,\ x_{33} = 160$	$z^* = 77600$	73,720 unit

optimal solution and total cost. The results are investigated through TORA optimization software.

6. Discussion of the Results

In the dynamic landscape of operations management, decision-makers face the challenge of navigating uncertainties, especially in the realm of transportation costs. The primary objective is to enhance decision-making by conducting comprehensive sensitivity analyses. We observed from **Table 6** that when we changed the values of parameters $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ and β_4 the optimal solution, $x_{12} = 130$, $x_{14} = 20$, $x_{21} = 140$, $x_{24} = 120$, $x_{32} = 270$ and $x_{33} = 250$, remained unchanged, but the total transportation cost changed by a significant amount $d = z^* - z$. And also, we observed that from **Table 9**, when we changed the values of parameters *k*, the optimal solution $x_{11} = 30$, $x_{12} = 170$, $x_{13} = 50$, $x_{21} = 100$ and $x_{33} = 160$ remain unchanged, but total transportation cost is increased by k times *i.e.*, $z^* = kz$. Significant changes in costs can have a profound impact on the optimal transportation plan. When costs decrease significantly, it may open up opportunities for cost savings and efficiency improvements. For example, if transportation costs increase, it may be necessary to allocate more resources or budget to transportation to maintain service levels. Conversely, if costs decrease, resources may be reallocated to other areas of the business. Transportation costs are closely tied to service quality and customer satisfaction. Significant cost changes can affect delivery times, shipping options, and overall service quality. Analyzing cost changes can help organizations align their transportation strategies with sustainability goals. The sensitivity analysis is vital tools in the real world for understanding and managing the impact of cost variations on transportation problems. These procedures enable organizations to make informed decisions, optimize resource allocation, and remain adaptable in a dynamic business environment.

7. Conclusion

In this paper, we have provided two approaches (theorems) for solving transportation problems and proved them in a simple and easy way. This method provides a detailed understanding of how adjustments to specific parameters influence the financial aspects of transportation, offering valuable insights for strategic decision-making in transportation management. To present a clear overview of the theorems introduced herein, two illustrative numerical examples have been selected to demonstrate the verification of the theorems. Different models may emphasize different aspects of the Transportation Problem. By using two models, analysts can gain a more comprehensive understanding of how variations in transportation costs impact the problem from different angles, considering numerical factors. The model can be developed for further study to effectively deal with unbalanced transportation problems.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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