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A NOVEL KIND OF GENERALIZED BELIEF INTERVAL-VALUED SOFT SET AND IT'S APPLICATION IN DECISION- MAKING

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ABSTRACT

In this paper, we introduce the novel concept of generalized belief interval-valued soft set (briefly, GBIVSS) and examine their properties. Also, we define the nine types of operations (for example, subset, equal, union, intersection, restricted union, extended intersection, complement, soft max-AND, and soft min-OR) on the GBIVSS. The basic theoretical of the above nine operations are given. Based on the concept of GBIVSS, we construct an algorithm to solve the soft decision-making problem, and extended their applicability with help of illustrative examples. Finally, a comparative analysis study has been constructed and compared with the result of Bashir et al.'s approach.

Keywords: Fuzzy set; Fuzzy parameter set; Interval-valued fuzzy; Belief interval-valued fuzzy set; Generalized belief interval-valued soft set; Decision-making.

1. INTRODUCTION

In 1999, Molodove [1] developed the new notion it's called 'soft set theory '. Recently, many authors (e.g., [2-25]) extended notions of soft sets and gave applications to solve the soft decision-making problems. In 2019, Vijayabalaji and Ramesh [26] proposed the concept of belief interval soft set and explain their properties. Cheng et al. [27] presented the notions of GBIVSS with application. In 2012, Wenqing et al. [28] introduced the notions of possibility belief interval-valued soft set and its application in decision-making. In this paper, by combining between the belief interval soft set and fuzzy parameter soft set, we will propose the concept of GBIVSS and its application in decision making. The operations of GBIVSS are defined and the basic properties are explained. Further, we suggest an algorithm (i.e., an application of a GBIVSS) to solve decision-making problem. Finally, we present a comparison between our proposed approach and the Bashir et. al's approach [29].

The rest of this article is arranged as follows. In section 2 it is mainly introduced several concepts related to fuzzy parameter set, interval-valued fuzzy soft set, belief interval-value soft set, soft set, fuzzy soft set, and belief interval-valued soft set. In section 3, is about GBIVSS, which containing their basic operations and structure properties. In Section 4 we explain an algorithm of GBIVSS for decision-making. Finally, in Section 5 conclusions are given.

2. PRELIMINARIES

We will provide a short survey of several required definitions in this paper as shown below.

2.1 Fuzzy parameter set. Khalil and Hassan [30] defined the fuzzy parameter set q in a parameter set L as follows:

Definition 2.1 A fuzzy parameter set q_l in a parameter set L is defined by $q_l : L \to [0, 1]$ $\forall l \in LL$, the membership value q_l basically determines the degree to which $l \in L$, belongs to the fuzzy parameter set $q_l(x)$ ($\forall x \in X$). The set of all fuzzy parameter sets will be denoted by $(I)^{XL}$

Now, we will introduce the notion of fuzzy parameter fuzzy parameter set as shown below.

Definition 2.2 Suppose that $q_l, p_l \in (I)^{\epsilon}$. Then, the intersection, the union and the complement are defined as

(1) $(q_l \cap p_l)(x) = q_l(x) \wedge p_l(x)$,

(2) $(q_l \cup p_l)(x) = q_l(x) \vee p_l(x)$, (3) $(q_l)^c(x) = 1 - q_l(x)$.

Where $q_l, p_l \in (I)^X$. By $q_l \subseteq p_l$, we mean that $q_l \leq p_l$ $\forall l \in L$. Clearly, if $q_l \subseteq p_l$ and $p_l \subseteq q_l$ then, $q_l = p_l$ $\forall l \in L$.

2.2. Interval-valued fuzzy set. We concept the notion of interval-valued fuzzy set [31] as follows: **Definition 2.3.** Suppose that Int([0, 1]) represent the set of all closed subintervals of [0, 1]. We call S be an interval-valued fuzzy set over a set X if S: X → Int([0, 1]) (∀x ∈ X , S(x)) can be written as

$$S(x) = \left\{\frac{x}{[s^-(x), s^+(x)]} \mid x \in \mathcal{X}\right\},$$

where $s^-(x)$ and $s^+(x)$ are referred to as the lower and upper degrees of membership an element x to X, respectively, and it satisfy $0 \leq s^-(x) \leq s^+(x) \leq 1$ ($\forall x \in X$).

Definition 2.4 Suppose that S and T be an interval-valued fuzzy sets over X, where

$$T(x) = \left\{\frac{x}{[t^-(x), t^+(x)]} \mid x \in \mathcal{X}\right\},$$

Then the basic operations (i.e. complement, union, and intersection) are defined as follows:

(1) The complement of S is denoted by S^c such that

$$S^c(x) = \left\{\frac{x}{[1-s^+(x), 1-s^-(x)]} \mid x \in \mathcal{X}\right\}.$$

(2) The union of S and T is denoted by S ∪ T such that

$$(S \cup T) = \left\{\frac{x}{[s^-(x) \vee t^-(x), s^+(x) \vee t^+(x)]} \mid x \in \mathcal{X}\right\}.$$

(3) The intersection of S and T is denoted by S ∩ T such that

$$(S \cap T) = \left\{\frac{x}{[s^-(x) \wedge t^-(x), s^+(x) \wedge t^+(x)]} \mid x \in \mathcal{X}\right\}.$$

2.3 Belief interval-value.

Desumpeter [20] proposed the notion of belief measure theory and Shafer [21] gave a thorough explanation of the belief function. In the following, we introduce several basic concepts of the theory of belief measure.

Definition 2.5 Suppose that X be a limited set of hypotheses (discrimination frame), 2^X all subset of X and $\breve{X} \subseteq X$. The structure of belief Dempster-Shafer has associated mapping M: $2^X \rightarrow [0, 1]$ such that a basic assignment function from subset of X into [0, 1] defined by, M $(\emptyset) = 0$, $\sum_{\breve{X} \in 2X} M(\breve{X}) = 1$. The focal elements that determinants are subsets of X are called non-zero values.

Definition 2.6 (1) Let Bel: $2^X \rightarrow [0, 1]$ be a measure of belief for $\breve{Y} \subseteq X$,

$$Bel\,(\breve{Y}) = \sum_{(\emptyset \neq \breve{X} \subseteq \breve{Y}} M(\breve{X}).$$

(2) Let Pl: $2^X \rightarrow [0, 1]$ be a measure of plausibility associated with M such that for any

$$\breve{Y} \subseteq X, Bel(Y) = \sum_{(\breve{X} \cap \breve{Y} \neq \emptyset} M(\breve{X}).$$

Clearly, $Bel(\tilde{Y}) \leq Pl(\tilde{Y})$. The interval $[Bel(\tilde{Y}), Pl(\tilde{Y})]$ called the belief interval.

2.1 Soft set, fuzzy soft set, and belief interval-valued soft set

Molodtsov [1] proposed a new approach to the so-called soft set theory and Maji et al. [6-8] introduced the concept of fuzzy soft sets.

An element $\tilde{\mathbb{F}}$ (traditionally, written as $(\tilde{\mathbb{F}}, \mathcal{L})$) of $[0,1]^{\mathcal{X}\mathcal{L}}$ (the set of all mappings from $\mathcal{L}$ to $[0,1]^{\mathcal{L}}$) is also called a fuzzy soft set over $\mathcal{L}$ indexed by the parameter set $\mathcal{L}$, where $[0,1]^{\mathcal{X}}$ is the set of all fuzzy subsets of $\mathcal{X}$. Vijayaballage and Ramesh suggested [26] the concept of belief interval-valued soft set by combining belief interval-valued (Dempster-Shafer Theory) and soft set and introduced several operations of belief interval-valued soft sets as shown below.

Definition 2.1 *Let $B^{\mathcal{X}}$ be a set of all belief interval-valued subsets of $\mathcal{X}$. A mapping $Q: \mathcal{L} \rightarrow B^{\mathcal{X}}$ is called a belief interval-valued soft set over $\mathcal{X}$, where $\mathcal{L}$ be a set of parameters.*

It's worth mentioning that for each parameter $\mathcal{L}, Q(l)$ can be written by

$$Q(\mathcal{L}) = \left\{\frac{x}{[Bel_{Q(l)}(x), Pl_{Q(l)}(x)]} \mid x \in \mathcal{X}\right\},$$

where, $Bel_{Q(l)}(x) \in [0,1]$, $Pl_{Q(l)}(x) \in [0,1]$, and $0 \leq Bel_{Q(l)}(x) \leq Pl_{Q(l)}(x) \leq 1$ $(\forall x \in \mathcal{X})$.

Definition 2.2 *Suppose that Q (i.e., $Q: \mathcal{L} \rightarrow B^{\mathcal{X}}$) $\in B^{\mathcal{X}}$ and P (i.e., $P: \mathcal{L} \rightarrow B^{\mathcal{X}}$). Then,*

(1) The intersection between Q and P, denoted by $D = Q \sqcap P$ (where $H: \mathcal{L} \rightarrow B^{\mathcal{X}}$ such that $\mathcal{L} = \mathcal{L}_1 \cap \mathcal{L}_2 \neq \emptyset, l \in \mathcal{L}$), is defined by

$$H(x)(l) = Q(l)(x) \cap P(l)(x)(\forall x \in \mathcal{X})$$

(2) The union between Q and P, denoted by $D = Q \sqcup P_p$ (where $D: \mathcal{L} \rightarrow B^{\mathcal{X}}$ such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2, l \in \mathcal{L}$), is defined by

$$D(l)(x) = \begin{matrix} Q(x) & , l \in \mathcal{L}_1 - \mathcal{L}_2 \\ Q(x) \cup P(x) & , l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ P(x) & , l \in \mathcal{L}_2 - \mathcal{L}_1 \end{matrix}$$

$(\forall x \in \mathcal{X})$,

Definition 2.3 *Suppose that Q (i.e., $Q_q: \mathcal{L} \rightarrow B^{\mathcal{X}}$). The complement Q of Q^c, is defined by*

$$Q_q^c = \{(\frac{x}{[1 - Pl_{Q(l)}(x), 1 - Bel_{Q(l)}(x)]}) | x \in \mathcal{X}, l \in \mathcal{L}\}$$

2 Generalized belief interval valued soft set

Definition 3.1 *Suppose that $\mathcal{X}$ (i.e., $\mathcal{X}$ be a set), $\mathcal{L}$ (i.e., $\mathcal{L}$ be a set of parameters), and $(\mathcal{X},\mathcal{L})$ be a soft universes. Let $Q:\mathcal{L}\to B^{\mathcal{X}}$ (i.e., $Q_{\mathcal{L}}$ is a belief interval-valued soft set [26]) and $q:\mathcal{L}\to[0,1]$ (i.e., $q_{\mathcal{L}}$ is a fuzzy parameter set [30]). If $Q_q:\mathcal{L}\to B^{\mathcal{X}}\times q_{\mathcal{L}}$ be a mapping, defined by*

$$Q_q(l)=(Q(l)(x),q(l)(x))(x\in\mathcal{X},l\in\mathcal{L})$$

Then Q_q is called a generalized belief interval valued soft set GBIVSS over $(\mathcal{X},\mathcal{L})$. Also, we can write $Q_q(l)$ as

$$Q_q=\{(\frac{x}{[Bel_{Q_q(l)}(x),Pl_{Q_q(l)}(x)]},q(l)(x))|x\in\mathcal{X}\}$$

The set of all GBIVSS, denoted by $(Ge)^{\mathcal{X}\mathcal{L}}$ or $(Ge^{\mathcal{X}})^{\mathcal{L}}$.

Example 3.2 *Assume that $\mathcal{X}=\{x_1,x_2,x_3\}$ be a set of three epochal laptops and $\mathcal{L}$ be a set of parameters containing l_1 represent 'speed', l_2 'color', and l_3 represent 'price' with fuzzy parameter $q_{\mathcal{L}}$. Then, $Q_q(l)(x)$ (i.e., $x_1,x_2,x_3\in\mathcal{X}$ and $l_1,l_2,l_3\in\mathcal{L}$) defined by*

$$Q_q(l_1)=\{(\frac{x_1}{[0.3,0.5]},0.2),(\frac{x_2}{[0.2,0.4]},0.8),(\frac{x_3}{[0.1,0.6]},0.7)\},$$

$$Q_q(l_2)=\{(\frac{x_1}{[0.2,0.5]},0.7),(\frac{x_2}{[0.1,0.6]},0.9),(\frac{x_3}{[0.1,0.8]},0.2)\},$$

$$Q_q(l_3)=\{(\frac{x_1}{[0.3,0.4]},0.8),(\frac{x_2}{[0.1,0.4]},0.7),(\frac{x_3}{[0.3,0.6]},0.5)\}.$$

Also, we can express $Q_q(l)(x)$ by the following matrix:

$$Q_q(l)(x)=\begin{pmatrix}([0.3,0.5],0.2) & ([0.2,0.4],0.8) & ([0.1,0.6],0.7)\\ ([0.2,0.5],0.7) & ([0.1,0.6],0.9) & ([0.1,0.8],0.2)\\ ([0.2,0.4],0.8) & ([0.1,0.4],0.7) & ([0.3,0.6],0.5)\end{pmatrix}.$$

Definition 3.3 *Suppose that Q_q (i.e., $Q_q:\mathcal{L}\to B^{\mathcal{X}}\times q_{\mathcal{L}})\in(Ge)^{\mathcal{X}\mathcal{L}}$ and P_p (i.e., $P_p:\mathcal{L}\to B^{\mathcal{X}}\times p_{\mathcal{L}})\in(Ge)^{\mathcal{X}\mathcal{L}}$. Then, Q_q is a subset P_p, denoted by $Q_q\subseteq P_p$ if*

(1) $q(l)(x)\le p(l)(x)$ $(\forall x\in\mathcal{X})$, and

(2) $Bel_{Q(\mathcal{L})}(x)\le Bel_{P(\mathcal{L})}(x)$ and $Pl_{Q(\mathcal{L})}(x)\le Pl_{P(\mathcal{L})}(x)$ $(\forall x\in\mathcal{X})$.

Example 3.4 *Let $\mathcal{X}=\{x_1,x_2,x_3\}$ be a set and $\mathcal{L}=\{l_1,l_2,l_3\}$ be a set of parameters and by Dentition 3.3, we have*

$$Q_q(l_1)=\{(\frac{x_1}{[0.3,0.5]},0.2),(\frac{x_2}{[0.2,0.4]},0.8),(\frac{x_3}{[0.1,0.6]},0.7)\},$$

$$Q_q(l_2)=\{(\frac{x_1}{[0.2,0.5]},0.1),(\frac{x_2}{[0.1,0.6]},0.5),(\frac{x_3}{[0.1,0.7]},0.3)\},$$

$$Q_q(l_3)=\{(\frac{x_1}{[0.2,0.4]},0.3),(\frac{x_2}{[0.1,0.4]},0.7),(\frac{x_3}{[0.3,0.6]},0.5)\}.$$

and

$$P_p(l_1)=\{(\frac{x_1}{[0.4,0.6]},0.3),(\frac{x_2}{[0.3,0.5]},0.9),(\frac{x_3}{[0.2,0.7]},0.9)\},$$

$$P_p(l_2) = \{(\frac{x_1}{[0.4, 0.7]}, 0.7), (\frac{x_2}{[0.3, 0.9]}, 0.7), (\frac{x_3}{[0.4, 0.9]}, 0.5)\},$$

$$P_p(l_3) = \{(\frac{x_1}{[0.4, 0.8]}, 0.5), (\frac{x_2}{[0.5, 0.6]}, 0.9), (\frac{x_3}{[0.4, 0.5]}, 0.7)\}.$$

Thus, $Q_q(\mathcal{L})(x) \subseteq P_p(\mathcal{L})(x)$ $(\forall x \in \mathcal{X})$.

Definition 3.5 *Suppose that* Q_q *(i.e.,* $Q_q: \mathcal{L} \to B^{\mathcal{X}} \times q_{\mathcal{L}}$*)* $\in (Ge)^{\mathcal{X}\mathcal{L}}$ *and* P_p *(i.e.,* $P_p: \mathcal{L} \to B^{\mathcal{X}} \times p_{\mathcal{L}}$*)* $\in (Ge)^{\mathcal{X}\mathcal{L}}$*. Then,* Q_q *is equal* P_p*, denoted by* $Q_q = P_p$ *if* $Q_q \subseteq P_p$ *and* $P_p \subseteq Q_q$.

Definition 3.6 *Suppose that* Q_q *(i.e.,* $Q_q: \mathcal{L} \to B^{\mathcal{X}} \times q_{\mathcal{L}}$*)* $\in (Ge)^{\mathcal{X}\mathcal{L}}$ *and* P_p *(i.e.,* $P_p: \mathcal{L} \to B^{\mathcal{X}} \times p_{\mathcal{L}}$*)* $\in (Ge)^{\mathcal{X}\mathcal{L}}$*. Then,*

(1) The intersection betweenQ_q and P_p, denoted by $D_d = Q_q \sqcap P_p$ (where $H_h: \mathcal{L} \to B^{\mathcal{X}} \times h_{\mathcal{L}}$ such that

$\mathcal{L} = \mathcal{L}_1 \cap \mathcal{L}_2 \neq \emptyset, l \in \mathcal{L}$), is defined by

$$H_h(l) = (H(l)(x), h(l)(x))(x \in \mathcal{X}, l \in \mathcal{L}),$$

where

$$H_h(x)(l) = Q_q(l)(x) \cap P_p(l)(x) \quad (x \in \mathcal{X}),$$

And $$h(l)(x) = \min(q(l)(x), p(l)(x)).$$

(2) The union between Q_q and P_p, denoted by $D_d = Q_q \sqcup P_p$ (where $D_d: \mathcal{L} \to B^{\mathcal{X}} \times d_{\mathcal{L}}$ such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2, l \in \mathcal{L}$), is defined by

$$D_d(l) = (D(l)(x), d(l)(x))(x \in \mathcal{X}, l \in \mathcal{L}), \text{ where}$$

$$D(l)(x) = \begin{cases} Q_q(x) & , l \in \mathcal{L}_1 - \mathcal{L}_2 \\ Q_q(x) \cup P_p(x) & , l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ P_p(x) & , l \in \mathcal{L}_2 - \mathcal{L}_1 \end{cases}.$$

$x \in \mathcal{X}$, and $d(l)(x) = \max(q(l)(x), p(l)(x))$.

(3) The extended intersection between Q_q and P_p, denoted by $Z_z = Q_q \sqcap_{ext} P_p$ (where $Z_z: \mathcal{L} \to B^{\mathcal{X}} \times z_{\mathcal{L}}$ such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2, l \in \mathcal{L}$), is defined by

$$Z_z(l) = (Z(l)(x), z(l)(x))(x \in \mathcal{X}, l \in \mathcal{L}),$$

where

$$Z(l)(x) = \begin{cases} Q_q(x) & , l \in \mathcal{L}_1 - \mathcal{L}_2 \\ Q_q(x) \cap P_p(x) & , l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ P_p(x) & , l \in \mathcal{L}_2 - \mathcal{L}_1 \end{cases}.$$

$x \in \mathcal{X}$, and $z(l)(x) = \max(q(l)(x), p(l)(x))$.

(4) The restricted union between Q_q and P_p, denoted by $W_w(x)(l) = Q_q \sqcup_{res} P_p$ (where $W_w: \mathcal{L} \to B^{\mathcal{X}} \times w_{\mathcal{L}}$ such that $\mathcal{L} = \mathcal{L}_1 \cap \mathcal{L}_2 \neq \emptyset, l \in \mathcal{L}$), is defined by

$$W_w(l) = (W(l)(x), w(l)(x))(x \in \mathcal{X}, l \in \mathcal{L}),$$

where

$$W_w(x)(l) = Q_q(l)(x) \cup P_p(l)(x) \quad (x \in \mathcal{X}),$$

and $w(l)(x) = \min(q(l)(x), p(l)(x))$.

Example 3.7 Let $\mathcal{X} = \{x_1, x_2, x_2\}$ be a set and $\mathcal{L} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$ be a set of parameters, where $\mathcal{L}_1 = \{l_1, l_2\}$ and $\mathcal{L}_2 = \{l_1, l_3\}$ be subset of $\mathcal{L}$. Then, Q_q (i.e., $Q_q: \mathcal{L}_1 \to B^{\mathcal{X}} \times q_{\mathcal{L}_1}$) $\in (Ge)^{\mathcal{X}\mathcal{L}}$ and P_p (i.e., $P_p: \mathcal{L}_2 \to B^{\mathcal{X}} \times p_{\mathcal{L}_2}$) $\in (Ge)^{\mathcal{X}\mathcal{L}}$ is given by

$$Q_q(\mathcal{L}_1) = \{(\frac{x_1}{[0.3, 0.5]}, 0.7), (\frac{x_2}{[0.4, 0.6]}, 0.5), (\frac{x_3}{[0.2, 0.7]}, 0.9)\},$$

$$Q_q(\mathcal{L}_2) = \{(\frac{x_1}{[0.5, 0.5]}, 0.3), (\frac{x_2}{[0.2, 0.4]}, 0.8), (\frac{x_3}{[0.3, 0.4]}, 0.8)\},$$

and,

$$P_p(\mathcal{L}_1) = \{(\frac{x_1}{[0.3, 0.5]}, 0.4), (\frac{x_2}{[0.3, 0.7]}, 0.1), (\frac{x_3}{[0.4, 0.5]}, 0.2)\},$$

$$P_p(\mathcal{L}_3) = \{(\frac{x_1}{[0.2, 0.4]}, 0.2), (\frac{x_2}{[0.2, 0.4]}, 0.3), (\frac{x_3}{[0.4, 0.4]}, 0.2)\}.$$

Thus, by Definition 3.6 we have

$$D_d(l_1) = \{(\frac{x_1}{[0.3, 0.5]}, 0.7), (\frac{x_2}{[0.4, 0.7]}, 0.5), (\frac{x_3}{[0.4, 0.7]}, 0.9)\},$$

$$D_d(l_2) = \{(\frac{x_1}{[0.5, 0.5]}, 0.3), (\frac{x_2}{[0.2, 0.4]}, 0.8), (\frac{x_3}{[0.3, 0.4]}, 0.8)\},$$

$$D_d(l_3) = \{(\frac{x_1}{[0.2, 0.4]}, 0.2), (\frac{x_2}{[0.2, 0.4]}, 0.3), (\frac{x_3}{[0.4, 0.4]}, 0.2)\},$$

and

$$H_h(\mathcal{L}_1) = \{(\frac{x_1}{[0.3, 0.2]}, 0.4), (\frac{x_2}{[0.3, 0.6]}, 0.1), (\frac{x_3}{[0.2, 0.5]}, 0.2)\}.$$

Theorem 3.8 *Suppose that* $B_b, C_c, D_d \in (Ge)^{\mathcal{X}\mathcal{L}}$. *The following hold:*

(1) $B_b = B_b \sqcap B_b$,

(2) $B_b = B_b \sqcup B_b$,

(3) $B_b \sqcap C_c = C_c \sqcap B_b$,

(4) $B_b \sqcup C_c = C_c \sqcup B_b$,

(5) $B_b \sqcap (C_c \sqcap D_d) = (B_b \sqcap C_c) \sqcap D_d$,

(6) $B_b \sqcup (C_c \sqcup D_d) = (B_b \sqcup C_c) \sqcup D_d$.

Proof. (1)-(4) are holding from Definition 3.6.

(5) Let C_c (i.e., $C_c: \mathcal{L}_1 \to B^{\mathcal{X}} \times c_{\mathcal{L}_1}$) $\in (Ge)^{\mathcal{X}\mathcal{L}}$ and D_d (i.e., $D_d: \mathcal{L}_2 \to B^{\mathcal{X}} \times d_{\mathcal{L}_2}$) $\in (Ge)^{\mathcal{X}\mathcal{L}}$. By Definition 3.6, we have $H_h = C_c \sqcap D_d$ (where $H_h: \mathcal{L} \to B^{\mathcal{X}} \times h_{\mathcal{L}}$ such that $\mathcal{L} = \mathcal{L}_1 \cap \mathcal{L}_2 \neq \emptyset$ and $h_{\mathcal{L}} = \min(c(l_1), d(l_2))$. Then $B_b \sqcap (C_c \sqcap D_d) = B_b \sqcap H_h$ (i.e., $B_b: \mathcal{L}_3 \to B^{\mathcal{X}} \times b_{\mathcal{L}_3}$) $\in (Ge)^{\mathcal{X}\mathcal{L}}$. Again, we suppose $K_k = B_b \sqcap H_h$ (where $K_k: \mathcal{L} \to B^{\mathcal{X}} \times k_{\mathcal{L}}$) such that $\mathcal{L} = \mathcal{L}_1 \cap \mathcal{L}_2 \cap \mathcal{L}_3$ and $k_{\mathcal{L}} = \min(c(l_1), d(l_2), b(l_3))$ $(\forall x \in \mathcal{X}, l \in \mathcal{L})$. So,

$$K_k = B_b \sqcup C_c \sqcup D_d \quad (1)$$

On the other side, let B_b (i.e., $B_b: \mathcal{L}_3 \to B^{\mathcal{X}} \times b_{\mathcal{L}_3}) \in (Ge)^{\mathcal{X}\mathcal{L}}$, and C_c (i.e.,$C_c: \mathcal{L}_1 \to B^{\mathcal{X}} \times c_{\mathcal{L}_1}$) $\in (Ge)^{\mathcal{X}\mathcal{L}}$. By Definition 3.6 we have $M_m = B_b \sqcap C_c$ (where $M_m: \mathcal{L} \to B^{\mathcal{X}} \times m_{\mathcal{L}}$ such that $\mathcal{L} = (\mathcal{L}_1 \cap \mathcal{L}_2)$ and $m_{\mathcal{L}} = \min(b(l_1), c(l_2))$. Then $(B_b \sqcap C_c) \sqcap D_d = M_m \sqcap D_d$. Again, we suppose $A_a = M_m \sqcap D_d$, (where $A_a: \mathcal{L} \to B^{\mathcal{X}} \times a_{\mathcal{L}}$) such that $\mathcal{L} = (\mathcal{L}_1 \cap \mathcal{L}_2 \cap \mathcal{L}_3)$ and $a_{\mathcal{L}} = \min(b(l_1), c(l_2), d(l_3))$ $(\forall x \in \mathcal{X}, l \in \mathcal{L})$. So,

$$A_a = B_b \sqcap C_c \sqcap D_d \quad (2)$$

From (1) and (2), we get on

$K_k(x) = A_a(x)$ $(\forall x \in \mathcal{X}, l \in \mathcal{L})$. So (5) is holding.

(6) is similar to (5).

Theorem 3.9 *Suppose that* $B_b, C_c, D_d \in (Ge)^{\mathcal{X}\mathcal{L}}$. *The following hold:*

(1) $B_b = B_b \sqcup_{res} B_b$,

(2) $B_b = B_b \sqcap_{ext} B_b$,

(3) $B_b \sqcup_{res} C_c = C_c \sqcup_{res} B_b$,

(4) $B_b \sqcap_{ext} C_c = C_c \sqcap_{ext} B_b$,

(5) $B_b \sqcap_{ext} (C_c \sqcap_{ext} D_d) = (B_b \sqcap_{ext} C_c) \sqcap_{ext} D_d$,

(6) $B_b \sqcup_{res} (C_c \sqcup_{res} D_d) = (B_b \sqcup_{res} C_c) \sqcup_{res} D_d$.

Proof. (1)-(4) are holding from Definition 3.6.

(5) Let C_c (i.e., $C_c: \mathcal{L}_1 \to B^{\mathcal{X}} \times c_{\mathcal{L}_1}) \in (Ge)^{\mathcal{X}\mathcal{L}}$ and D_d (i.e., $D_d: \mathcal{L}_2 \to B^{\mathcal{X}} \times d_{\mathcal{L}_2}) \in (Ge)^{\mathcal{X}\mathcal{L}}$. By Definition 3.6, we have $H_h = C_c \sqcap_{ext} D_d$ (where $H_h: \mathcal{L} \to B^{\mathcal{X}} \times h_{\mathcal{L}}$ such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2$ and $h_{\mathcal{L}} = \max(c(l_1), d(l_2))$. Then

$$H(x)(l) = \begin{cases} C_c & , l \in \mathcal{L}_1 - \mathcal{L}_2 \\ C_c \cap D_d & , l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ D_d & , l \in \mathcal{L}_2 - \mathcal{L}_1 \end{cases}.$$

$x \in \mathcal{X}$. As $B_b \sqcap_{ext} (C_c \sqcap_{ext} D_d) = B_b \sqcap_{ext} H_h$ (i.e., $B_b: \mathcal{L}_3 \to B^{\mathcal{X}} \times b_{\mathcal{L}_3}) \in (Ge)^{\mathcal{X}\mathcal{L}}$. Again,

we suppose $K_k = B_b \sqcap_{ext} H_h$ (where $K_k: \mathcal{L} \to B^{\mathcal{X}} \times k_{\mathcal{L}}$) such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2 \cup \mathcal{L}_3$ and

$k_{\mathcal{L}} = \max(c(l_1), d(l_2), b(l_3))$ $(\forall x \in \mathcal{X}, l \in \mathcal{L})$. Then

$$K(x)(l) = \begin{cases} B_b(x)(l) & , l \in \mathcal{L}_1 - (\mathcal{L}_2 - \mathcal{L}_3) \\ C_c(x)(l) & , l \in \mathcal{L}_2 - (\mathcal{L}_1 - \mathcal{L}_3) \\ D_d(x) & , l \in \mathcal{L}_3 - (\mathcal{L}_1 - \mathcal{L}_2) \\ C_c(x) \cap D_d(x) & , l \in \mathcal{L}_2 \cap (\mathcal{L}_3 - \mathcal{L}_1) \\ B_b(x) \cap C_c(x) & , l \in \mathcal{L}_1 \cap (\mathcal{L}_2 - \mathcal{L}_3) \\ B_b(x) \cap D_d(x) & , l \in \mathcal{L}_1 \cap (\mathcal{L}_3 - \mathcal{L}_2) \\ B_b(x) \cap C_c(x) \cap D_d(x) & , l \in \mathcal{L}_1 \cap (\mathcal{L}_2 \cap \mathcal{L}_3) \end{cases}.$$

So, $K_k = B_b \sqcap_{ext} C_c \sqcap_{ext} D_d$ (3)

On the other side, let B_b (i.e., $B_b:\mathcal{L}_3 \to B^{\mathcal{X}} \times b_{\mathcal{L}_1}) \in (Ge)^{\mathcal{X}\mathcal{L}}$, and C_c (i.e.,$C_c:\mathcal{L}_1 \to B^{\mathcal{X}} \times c_{\mathcal{L}_1}$) $\in (Ge)^{\mathcal{X}\mathcal{L}}$. By **Definition 3.9** we have $M_m = B_b \sqcap_{ext} C_c$ (where $M_m:\mathcal{L} \to B^{\mathcal{X}} \times m_{\mathcal{L}}$ such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2$ and $m_{\mathcal{L}} = \max(b(l_1), c(l_2))$. Then

$$M(x)(l) = \begin{cases} B_b & ,l \in \mathcal{L}_1 - \mathcal{L}_2 \\ B_b \cup C_c & ,l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ C_c & ,l \in \mathcal{L}_2 - \mathcal{L}_1 \end{cases}.$$

$x \in \mathcal{X}$. As $B_b \sqcap_{ext} (C_c \sqcap_{ext} D_d) = M_m \sqcap_{ext} D_d$. Again, we suppose $A_a = M_m \sqcap_{ext} D_d$ (where $A_a:\mathcal{L} \to B^{\mathcal{X}} \times a_{\mathcal{L}}$) such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2 \cup \mathcal{L}_3$ and $a_{\mathcal{L}} = \max(c(l_1), d(l_2), b(l_3))$ $(\forall x \in \mathcal{X}, l \in \mathcal{L})$. Then

$$A(x)(l) = \begin{cases} B_b(x)(l) & ,l \in \mathcal{L}_1 - (\mathcal{L}_2 - \mathcal{L}_3) \\ C_c(x)(l) & ,l \in \mathcal{L}_2 - (\mathcal{L}_1 - \mathcal{L}_3) \\ D_d(x) & ,l \in \mathcal{L}_3 - (\mathcal{L}_1 - \mathcal{L}_2) \\ C_c(x) \cap D_d(x) & ,l \in \mathcal{L}_2 \cap (\mathcal{L}_3 - \mathcal{L}_1) \\ B_b(x) \cap C_c(x) & ,l \in \mathcal{L}_1 \cap (\mathcal{L}_2 - \mathcal{L}_3) \\ B_b(x) \cap D_d(x) & ,l \in \mathcal{L}_1 \cap (\mathcal{L}_3 - \mathcal{L}_2) \\ B_b(x) \cap C_c(x) \cap D_d(x) & ,l \in \mathcal{L}_1 \cap (\mathcal{L}_2 \cap \mathcal{L}_3) \end{cases}.$$

So,

$$A_a = B_b \sqcap_{ext} C_c \sqcap_{ext} D_d \quad (4)$$

From (3) and (4) we noted that

$$K_k(x) = A_a(x)$$

$(\forall x \in \mathcal{X}, l \in \mathcal{L})$. So (5) is holding.

(6) is similar to (5).

Definition 3.10 *Suppose that* Q_q*(i.e.,* $Q_q:\mathcal{L} \to B^{\mathcal{X}} \times q_{\mathcal{L}}$*)* $(Ge)^{\mathcal{X}\mathcal{L}}$*. The complement* Q_q *of* Q_q^c*, is defined by*

$$Q_q^c = \{(\frac{x}{[1-Pl_{Q(l)}(x), 1-Bel_{Q(l)}(x)]}, 1 - q(l)(x)) | x \in \mathcal{X}, l \in \mathcal{L}\}$$

Example 3.11 *(Continued from Example 3.2), The complement* Q_q^c *is computed by*

$$Q_q^c(l_1) = \{(\frac{x_1}{[0.5,0.7]}, 0.8), (\frac{x_2}{[0.6,0.8]}, 0.2), (\frac{x_3}{[0.4,0.9]}, 0.3)\},$$

$$Q_q^c(l_2) = \{(\frac{x_1}{[0.5,0.8]}, 0.3), (\frac{x_2}{[0.4,0.9]}, 0.1), (\frac{x_3}{[0.2,0.9]}, 0.8)\},$$

$$Q_q^c(l_3) = \{(\frac{x_1}{[0.6,0.7]}, 0.2), (\frac{x_2}{[0.6,0.9]}, 0.3), (\frac{a_3}{[0.4,0.7]}, 0.5)\}.$$

Theorem 3.12 *Suppose that* $B_b, C_c \in (Ge)^{\mathcal{X}\mathcal{L}}$*. The following hold:*

(1) $[B_b \sqcap_{ext} C_c]^c = B_b^c \sqcup C_c^c$,

(2) $[B_b \sqcup C_c]^c = B_b^c \sqcap_{ext} C_c^c$,

(3) $[B_b \sqcap C_c]^c = B_b^c \sqcup_{res} C_c^c$,

(4) $[B_b \sqcup_{res} C_c]^c = B_b^c \sqcap C_c^c$.

Proof. (1) Let B_b (i.e., $B_b: \mathcal{L}_1 \to B^{\mathcal{X}} \times b_{\mathcal{L}_1}) \in (Ge)^{\mathcal{X}\mathcal{L}}$ and C_c (i.e., $C_c: \mathcal{L}_2 \to B^{\mathcal{X}} \times c_{\mathcal{L}_2}) \in (Ge)^{\mathcal{X}\mathcal{L}}$. By Definition 3.6 (4) we have $D_d = B_b \sqcap_{ext} C_c$, where $D_d: \mathcal{L} \to B^{\mathcal{X}} \times d_{\mathcal{L}}$

$$D(l)(x) = \begin{cases} B_b(l)(x) & , l \in \mathcal{L}_1 - \mathcal{L}_2 \\ B_b(l)(x) \cap C_c(l)(x) & , l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ C_c(l)(x) & , l \in \mathcal{L}_2 - \mathcal{L}_1 \end{cases}.$$

such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2$ for all $l \in \mathcal{L}$, $\forall x \in \mathcal{X}$, and $d_{\mathcal{L}} = \max(b_{\mathcal{L}_1}, c_{\mathcal{L}_2})$. Then, by Definition 3.10 $[B_b \sqcap_{ext} C_c]^c = D_d^c$

$$D^c(l)(x) = \begin{cases} B_b^c(l)(x) & , l \in \mathcal{L}_1 - \mathcal{L}_2 \\ B_b^c(l)(x) \cup C_c^c(k)(x) & , l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ C_c^c(l)(x) & , l \in \mathcal{L}_2 - \mathcal{L}_1 \end{cases}.$$

$\forall l \in \mathcal{L}$, $\forall x \in \mathcal{X}$, and $1 - d_{\mathcal{L}} = \max(1 - b_{\mathcal{L}_1}, 1 - c_{\mathcal{L}_2})$ (5)

On the other side, by Definition 3.6 (1) and Definition 3.10, we have $E_e = B_b^c \sqcup C_c^c$, where $E_e: \mathcal{L} \to B^{\mathcal{X}} \times e_{\mathcal{L}}$

$$E(l)(x) = \begin{cases} B_b^c(l)(x) & , l \in \mathcal{L}_1 - \mathcal{L}_2 \\ B_b^c(l)(x) \cup C_c^c(l)(x) & , l \in \mathcal{L}_1 \cap \mathcal{L}_2 \\ C_c^c(l)(x) & , l \in \mathcal{L}_2 - \mathcal{L}_1 \end{cases}.$$

such that $\mathcal{L} = \mathcal{L}_1 \cup \mathcal{L}_2$ $\forall l \in \mathcal{L}$, $x \in \mathcal{X}$ and $e_{\mathcal{L}} = \max((1 - b_{\mathcal{L}_1}, 1 - c_{\mathcal{L}_2})$ (6)

Then, from (5) and (6) we get on $D_d^c(l)(x) = E_e^c(l)(x)$ $(\forall l \in \mathcal{L}, x \in \mathcal{X})$. Hence, (1) is holding.

(2)-(4) are similar to (1).

Definition 3.13 *Suppose that* Q_q*(i.e.,* $Q_q: \mathcal{L}_i \to B^{\mathcal{X}} \times q_{\mathcal{L}_i}$*)* $(Ge)^{\mathcal{X}\mathcal{L}}$ *and* P_p*(i.e.,* $P_p: \mathcal{L}_i \to B^{\mathcal{X}} \times p_{\mathcal{L}_j}$*)* $(Ge)^{\mathcal{X}\mathcal{L}}$*. Then,*

(1) The soft max-AND operation between Q_q and P_p, denoted by $D_d = Q_q \barwedge P_p$ (where $D_d: \mathcal{L}_i \times \mathcal{L}_j \to B^{\mathcal{X}} \times d_{(\mathcal{L}_i, \mathcal{L}_j)}$) is defined by

$$D_d = \{(\frac{x}{[Bel_{D_{d(l_i,l_j)}}(x), Pl_{D_{d(l_i,l_j)}}(x)]}, d(l_i, l_j)(x)) | x \in \mathcal{X}\},$$

where $[Bel_{D_{d(l_i,l_j)}}(x), Pl_{D_{d(l_i,l_j)}}(x)]$

$$= ([\tfrac{2}{3}(Bel_{Q_q(l_i)}(x) + Bel_{P_p(l_j)}(x)) - \tfrac{1}{3}\max\{Bel_{Q_q(l_i)}(x), Bel_{P_p(l_j)}(x)\},$$
$$\tfrac{2}{3}(Pl_{Q_q(l_i)}(x) + Pl_{P_p(l_j)}(x)) - \tfrac{1}{3}\max\{Pl_{Q_q(l_i)}(x), Pl_{P_p(l_j)}(x)\}])$$

and

$$d(l_i, l_j)(x) = \max(q_{\mathcal{L}_i}, p_{\mathcal{L}_j})$$

(2) The soft min-OR operation between Q_q and P_p denoted by $H_h = Q_q \veebar P_p$ (where, $H_h: \mathcal{L}_i \times \mathcal{L}_j \to B^{\mathcal{X}} \times h_{(\mathcal{L}_i \mathcal{L}_j)}$) is defined by

$$H_h = \{(\frac{x}{[Bel_{D_d(l_i,l_j)}(x), Pl_{D_d(l_i,l_j)}(x)]}, h(l_i, l_j)(x)) \mid x \in \mathcal{X}\},$$

where

$$\left[Bel_{H_h(l_i,l_j)}(x), Pl_{H_h(l_i,l_j)}(x)\right] = ([\tfrac{2}{3}(Bel_{Q_q(l_i)}(x) + Bel_{P_p(l_j)}(x)) - \tfrac{1}{3}\min\{Bel_{Q_q(l_i)}(x), Bel_{P_p(l_j)}(x)\}, \tfrac{2}{3}(Pl_{Q_q(l_i)}(x) + Pl_{P_p(l_j)}(x)) - \tfrac{1}{3}\min\{Pl_{Q_q(l_i)}(x), Pl_{P_p(l_j)}(x)\}])$$

and

$$h(l_i, l_j)(x) = \min(q_{\mathcal{L}_i}, p_{\mathcal{L}_j})$$

Example 3.14 *(Continued from Example 3.4) (1) The soft max-AND operation* $D_d = Q_q \barwedge P_p$. *Then, from Definition 3.12 (1) computed by*

$$D_d(\mathcal{L}_1) = \{(\frac{x_1}{[0.3,0.5]}, 0.3), (\frac{x_2}{[0.2,0.4]}, 0.9), (\frac{x_3}{[0.1,0.6]}, 0.9)\},$$

$$D_d(\mathcal{L}_2) = \{(\frac{x_1}{[0.2,0.5]}, 0.7), (\frac{x_2}{[0.1,0.7]}, 0.7), (\frac{x_3}{[0.2,0.7]}, 0.5)\},$$

$$D_d(\mathcal{L}_3) = \{(\frac{x_1}{[0.2,0.5]}, 0.9), (\frac{x_2}{[0.2,0.4]}, 0.9), (\frac{x_3}{[0.3,0.5]}, 0.7)\}.$$

(2) The soft min-OR operation $H_h = Q_q \veebar P_p$. Then, from Definition 3.12 (2) computed by

$$H_h(\mathcal{L}_1) = \{(\frac{x_1}{[0.4,0.5]}, 0.2), (\frac{x_2}{[0.2,0.4]}, 0.8), (\frac{x_3}{[0.1,0.6]}, 0.7)\},$$

$$H_h(\mathcal{L}_2) = \{(\frac{x_1}{[0.3,0.6]}, 0.1), (\frac{x_2}{[0.2,0.8]}, 0.5), (\frac{x_3}{[0.3,0.8]}, 0.3)\},$$

$$H_h(\mathcal{L}_3) = \{(\frac{x_1}{[0.3,0.6]}, 0.3), (\frac{x_2}{[0.3,0.5]}, 0.7), (\frac{x_3}{[0.3,0.5]}, 0.5)\}.$$

4 Application of generalized belief interval valued soft set in decision-making

In this section, we will present a novel method on generalized belief interval valued soft set for soft decision-making.

Example 4.1 Let $\mathcal{X} = \{x_1, x_2, \ldots, x_r\}$ be a set of element, $\mathcal{L} = \{l_1, l_2, \ldots, l_i\}$ be a set of parameters (where r and i are natural numbers), and $Q_q \in (Ge)^{\mathcal{X}\mathcal{L}}$ (i.e., $Q_q: \mathcal{L}_i \to B^{\mathcal{X}} \times q_{\mathcal{L}_i}$) is defining as follows:

$$Q_q(l_i) = \{(\frac{x_r}{[Bel_{Q_q(l_i)}(x_r), Pl_{Q_q(l_i)}(x_r)]}, q(l_i)(x_r)) \mid x_r \in \mathcal{X}\}, \text{ where}$$

$0 \le Bel_{Q_q(l_i)}(x_r) \le Pl_{Q_q(l_i)}(x_r) \le 1$.

Now, we will define two operations as follows (which we can reduce $Q_q(l_i)$ to $S_s(l_i)$ (i.e., belief interval valued soft set):

(1) $\psi(l_i)(x_r) = Pl_{Q_q(l_i)}(x_r) \times q(l_i)(x_r)$.

(2) $\xi(l_i)(x_r) = (Bel_{Q_q(l_i)}(x_r) + q(l_i)(x_r)) - Bel_{Q_q(l_i)}(x_r) \times q(l_i)(x_r). \forall x_r \in \mathcal{X}, 0 \le \psi(l_i)(x_r) \le \xi(l_i)(x_r) \le 1$,

such that

$$S_s(l_i) = \{(\frac{x_r}{[\psi(l_i)(x_r), \xi(l_i)(x_r)]}) \mid x_r \in \mathcal{X}\},$$

where $0 \le \psi(l_i)(x_r) \le \xi(l_i)(x_r) \le 1$.

Next, we present the below algorithm.

Algorithm

Step 1. Input $Q_q \in (Ge)^{\mathcal{X}\mathcal{L}}$,

$$Q_q(l_i) = \{(\frac{x_r}{[Bel_{Q_q(l_i)}(x_r), Pl_{Q_q(l_i)}(x_r)]}, q(l_i)(x_r)) | x_r \in \mathcal{X}\}.$$

Step 2. Reduce $Q_q(l_i)$ to $S_s(l_i)$ (i.e., belief interval valued soft set)) as

$$S_s(l_i) = \{\frac{x_r}{[\psi(l_i)(x_r), \xi(l_i)(x_r)]} | x_r \in \mathcal{X}\},$$

where $0 \le \psi(l_i)(x_r) \le \xi(l_i)(x_r) \le 1$.

Step3. Compute the choice value $\mathbb{C}_r$ such that $\mathbb{C}_r = [\mathbb{C}_r^{\psi}, \mathbb{C}_r^{\xi}] = [\sum_{l_i \in \mathcal{L}} \psi(l_i)(x_r), \sum_{l_i \in \mathcal{L}} \xi(l_i)(x_r)]$ for all $l_i \in \mathcal{L}$ and $x_r \in \mathcal{X}$.

Step4. Compute the score $\mathbb{R}_i$ which is calculated by $\mathbb{R}_i = \sum [\mathbb{C}_{ri}^{\psi} - \mathbb{C}_{rj}^{\psi}] + [\mathbb{C}_{ri}^{\xi} - \mathbb{C}_{rj}^{\xi}]$ for all $i, j = 1,2, \ldots, n$.

Step5. Determine the best choice

$$x_r = \max_{x_r \in \mathcal{X}} \{\mathbb{R}_i\}.$$

The following example, we show that the principal and steps of the approach to decision-making proposed in our paper.

Example 4.2 *Assume that there is $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$ be a set of four applicants for a job in a large company and this company requires a set of four parameters for applicants, where $\mathcal{L}$ be a set of parameters containing l_1 represent the experience factor, l_2 represent presentable, l_3 represent the language factor, and l_4 represent work under pressure. By above algorithm we will obtain the following.*

Step 1. The evaluation date can be given by $Q_q \in (Ge)^{\mathcal{X}\mathcal{L}}$ as

$$Q_q(l_1) = \{(\frac{x_1}{[0.2,0.4]}, 0.5), (\frac{x_2}{[0.3,0.7]}, 0.4), (\frac{x_3}{[0.5,0.5]}, 0.3), (\frac{x_4}{[0.2,0.3]}, 0.6)\},$$

$$Q_q(l_2) = \{(\frac{x_1}{[0.1,0.9]}, 0.8), (\frac{x_2}{[0.4,0.5]}, 0.7), (\frac{x_3}{[0.2,0.7]}, 0.5), (\frac{x_4}{[0.1,0.8]}, 0.5)\},$$

$$Q_q(l_3) = \{(\frac{x_1}{[0.4,0.5]}, 0.9), (\frac{x_2}{[0.4,0.6]}, 0.2), (\frac{x_3}{[0.4,0.9]}, 0.3), (\frac{x_4}{[0.3,0.6]}, 0.1)\},$$

$$Q_q(l_4) = \{(\frac{x_1}{[0.3,0.7]}, 0.1), (\frac{x_2}{[0.2,0.6]}, 0.1), (\frac{x_3}{[0.4,0.4]}, 0.6), (\frac{x_4}{[0.3,0.8]}, 0.4)\}.$$

Step 2. By computing $\psi(l_i)(x_r)$ and $\xi(l_i)(x_r)$ $(\forall i, r = \{1,2,3,4\})$ as follows:

$\psi(l_1)(x_1) = 0.4 \times 0.5 = 0.2$, $\xi(l_1)(x_1) = (0.2 + 0.5) - 0.2 \times 0.5 = 0.6$.

By the same way, we get on the following Table 1.

Table 1: $\psi(l_i)(x_r)$ and $\xi(l_i)(x_r)$

$\mathcal{L}$	l_1	l_2	l_3	l_4
x_1	[0.20, 0.60]	[0.72, 0.82]	[0.45, 0.94]	[0.18, 0.68]

x_2	[0.28, 0.58]	[0.35, 0.82]	[0.12, 0.52]	[0.40, 0.55]
x_3	[0.15, 0.65]	[0.35, 0.60]	[0.09, 0.46]	[0.06, 0.64]
x_4	[0.07, 0.37]	[0.18, 0.44]	[0.24, 0.76]	[0.32, 0.58]

Then, we compute the $S_s(x_r)$ (where $r = 1,2,3,4$) as

$$S_s(l_1) = \{\frac{x_1}{[0.20,0.60]}, \frac{x_2}{[0.28,0.58]}, \frac{x_3}{[0.15,0.65]}, \frac{x_4}{[0.18,0.68]}\},$$

$$S_s(l_2) = \{\frac{x_1}{[0.72,0.82]}, \frac{x_2}{[0.35,0.82]}, \frac{x_3}{[0.35,0.60]}, \frac{x_4}{[0.40,0.55]}\},$$

$$S_s(l_3) = \{\frac{x_1}{[0.45,0.94]}, \frac{x_2}{[0.12,0.52]}, \frac{x_3}{[0.09,0.46]}, \frac{x_4}{[0.06,0.64]}\},$$

$$S_s(l_4) = \{\frac{x_1}{[0.07,0.37]}, \frac{x_2}{[0.18,0.44]}, \frac{x_3}{[0.24,0.76]}, \frac{x_4}{[0.32,0.58]}\}.$$

Step 3. By computing the choice value $\mathbb{C}_r$ as shown in Table 2:

Table 2: The choice value $\mathbb{C}_r$

$\mathcal{L}$	l_1	l_2	l_3	l_4	$\mathbb{C}_r$
x_1	[0.20, 0.60]	[0.72, 0.82]	[0.45, 0.94]	[0.18, 0.68]	$\mathbb{C}_1$=[1.35, 3.04]
x_2	[0.28, 0.58]	[0.35, 0.82]	[0.12, 0.52]	[0.40, 0.55]	$\mathbb{C}_2$=[1.15, 2.47]
x_3	[0.15, 0.65]	[0.35, 0.60]	[0.09, 0.46]	[0.06, 0.64]	$\mathbb{C}_3$=[0.65, 2.35]
x_4	[0.07, 0.37]	[0.18, 0.44]	[0.24, 0.76]	[0.32, 0.58]	$\mathbb{C}_4$=[0.81,2.15]

Step 4. By computing the score result of $\mathbb{R}_i$, we get

$\mathbb{R}_1 = (1.35 - 1.35 + 3.04 - 3.04 + 1.35 - 1.15 + 3.04 - 2.47 + 1.35 - 0.65 + 3.04 - 2.35 + 1.3$
$3.04 - 2.15) = 3.59.$

Similarity, $\mathbb{R}_2 = 0.51, \mathbb{R}_3 = -1.97$ and

$\mathbb{R}_4 = -2.13.$

Step 5. By Step 4, the best choice is x_1 .

5. Comparison with existing works

In this subsection, we will compare the proposed approach and Bashir et al.'s approach [29] based on generalized belief interval valued soft set.

Example 5.2 *([29, see Section 6]) Suppose that there are three schools in universe $\mathcal{X} = \{x_1, x_2, x_3\}$ and the parameter set $\mathcal{L} = \{l_1, l_2, l_3, l_4, l_5, l_6\}$ which indicates a specific criterion for the schools l_1 stands for international, l_2 stands for English, l_3 stands for high efficiency, l_4 stands for modern, l_5 stands for full day, and l_6 stands for half day.*

Step 1. Now the evaluation data of possibility intuitionistic fuzzy soft set (briefly, PIFSS) [29] given as:

$$Q_q(l_1) = \{(\frac{x_1}{(0.3,0.5)}, 0.1), (\frac{x_2}{(0.5,0.3)}, 0.4), (\frac{x_3}{(0.3,0.4)}, 0.5)\},$$

$$Q_q(l_2) = \{(\frac{x_1}{(0.0,0.1)}, 0.9), (\frac{x_2}{(0.1,0.0)}, 0.3), (\frac{x_3}{(0.9,0.1)}, 0.2)\},$$

$$Q_q(l_3) = \{(\frac{x_1}{(0.6,0.3)}, 0.4), (\frac{x_2}{(0.5,0.4)}, 0.3), (\frac{x_3}{(0.6,0.3)}, 0.1)\}.$$

Step 2. Then by step 1, we can transform PIFSS [29] to GBVSS as.

$$S_s(l_1) = \{\frac{x_1}{[0.05,0.37]}, \frac{x_2}{[0.12,0.78]}, \frac{x_3}{[0.20,0.65]}\},$$

$$S_s(l_2) = \{\frac{x_1}{[0.08,0.00]}, \frac{x_2}{[0.00,1.00]}, \frac{x_3}{[0.02,0.92]}\},$$

$$S_s(l_3) = \{\frac{x_1}{[0.12,0.76]}, \frac{x_2}{[0.12,0.65]}, \frac{x_3}{[0.03,0.64]}\}.$$

Satisfy, $0 \leq Bel_{S_{s(l_1)}}(x) \leq Pl_{S_{s(l_1)}}(x) \leq 1 \ \ (\forall x \in \mathcal{X})$.

Step 3. By computing the choice value $\mathbb{C}_r$, we get on the following Table 3.

Table 3: The choice value $\mathbb{C}_r$

$\mathcal{L}$	l_1	l_2	l_3	$\mathbb{C}_r$
x_1	[0.05,0.37]	[0.12,0.78]	[0.20,0.65]	$\mathbb{C}_1$=[0.52,1.13]
x_2	[0.08,0.00]	[0.00,1.00]	[0.02,0.92]	$\mathbb{C}_2$=[0.24,1.84]
x_3	[0.12,0.76]	[0.12,0.65]	[0.03,0.64]	$\mathbb{C}_3$=[0.25,2.21]

Step 4. By computing the score result of $\mathbb{R}_i$, we get

$$\mathbb{R}_1 = (1.13 - 1.84 + 1.13 - 2.21 + 0.25 - 0.24 + 0.25 - 0.25) = -1.78.$$

Similarity, we can get $\mathbb{R}_2 = 0.32$ and $\mathbb{R}_3 = 1.46$.

Step 5. By Step 4, the best choice is x_3.

The comparison between our proposed approach and Bashir et al.'s approach [29] is presented by the following Table 4.

Table 4: The comparison between our proposed and Bashir et al.'s approach

	Ranking	Best Choice
Bashir et al.'s approach [29]	$x_2 \succ x_1 \succ x_3$	x_2
Our approach	$x_3 \succ x_2 \succ x_1$	x_3

By above Table 3, we notice that there is a difference between the decision in our proposed approach and Bashir et al.`s approach [29] that relied on the comparison table (i.e., there is an error in relying on the comparison table [32]). Therefore it becomes clear that our decision is the correct one. However, our proposed decision-making approach can successfully avoid and solve the above issues.

6 CONCLUSION

In this paper, we introduced a new concept which is generalized belief interval valued soft set and its application in the decision-making problem. First we present the concept of basic properties, and also explain the properties of GBIVSS. After that, we developed an algorithm to solve decision-making problem on GBIVSS and illustrated it's applicability throw a nonmoral.

In the future, we seek to extend each of these papers[33-38] and reviews into different mathematical applications.

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مفهوم جديد للقيمة اللينة لفترات الاعتقاد المعمم وتطبيقاتها فى اتخاذ القرار

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الملخص :

نبذة مختصرة. في هذه الورقة ، نقدم مفهومًا جديدًا للقيمة اللينة لفترات الاعتقاد المعمم و خصائصها. نحدد الأنواع التسعة من العمليات علي المجموعة و يتم إعطاء النظريات الأساسية واثباتها (على سبيل المثال ، مجموعة فرعية ، متساوية ، اتحاد ، تقاطع ، اتحاد مقيد و تقاطع ممتد). ومن ثم نقوم ببناء خوارزمية لحل مشكلة اتخاذ القرار استنادا علي توسيع نطاق المجموعه المعممة وتطبيقاتها وذللك عن طريق وضع امثلة توضيحية. أحيراً نقوم بدراسة تحليلية عن طريق مقارنة نتائج نهجنا المتبع ونهج آخر (بشير وآخرون).