An Analytical Solution of the Hyperbolic Bioheat Model of the Cornea Subjected to Laser Irradiation

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Abstract. The thermal effects occurring in a cornea subjected to short-pulsed laser irradiation during laser thermokeratoplasty (LTK) are investigated. The transient bioheat transfer is described by the hyperbolic model and solved analytically using the finite Fourier transform technique and the method of variation of parameters. The computational results predicted by the hyperbolic model show that the degree of damage in the corneal tissue induced by Ho: YAG laser irradiation under LTK surgery increases linearly with time. An increase in the convection coefficient of the anterior corneal surface causes an insignificant reduction in the corneal temperature, whereas an increase in the value of phase lag in the heat flux vector causes a rise in the corneal temperature. © 2022 Journal of Biomedical Photonics & Engineering.

Keywords: temperature; laser; cornea; refractive error; LTK; Finite Fourier transform technique; hyperbolic model.

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1 Introduction

The estimation of the temperature in biological tissues is under the focus of researchers in recent years. This fact is important in some clinical surgery such as radiofrequency heating of tissues, laser heating of tissues, tumour therapy or some skin treatment such as tattoos [1]. In ophthalmology, laser thermal shrinkage of collagenous tissue has been used clinically and experimentally for several procedures, including laser thermokeratoplasty (LTK). The surgical outcome of many of these procedures suffers from limited predictability and uncertain long-term stability, mainly because of a poor understanding of the fundamental thermodynamic and biomechanical processes of laser-induced thermal shrinkage of collagenous tissue. A better understanding of the thermo-optical response of the cornea and the kinetics of thermal shrinkage could help to optimize the laser treatment parameters for LTK and other procedures relying on thermal shrinkage [2], and perhaps improve the stability and predictability of the outcome.

In recent years, several bioheat transfer models for temperature rise in biological tissues have been developed and simulated. Hobiny and Abbas [3], Alzahrani and Abbas [4]; and Abbas, Hobiny, and Alzaharani [5] have used the Laplace transformation technique to obtain analytical solutions to their hyperbolic bioheat transfer models under different conditions, while Hobiny and Abbas [6] have used the finite element approach to solve their bioheat model. Ghanami and Abbas [7], Hobiny and Abbas [8]; and Hobiny et al. [9] developed Fractional bioheat transfer models to determine the temperature and the thermal damage in the tissue and obtained analytical solutions to the models using the Laplace transform. In 2020, Hobiny and Abbas [10] used the finite element approach to find a numerical solution to their nonlinear dual phase lag bioheat model for the temperature changes and evaluated the thermal damage using the Arrhenius equation. In these works, it has been concluded that mathematical models are efficient tools to evaluate the bioheat transfer in living tissues.

In 2008, Ooi et al. [11] developed a boundary element model of the human eye under LTK surgery using classical heat diffusion equation for heat transfer and the Beer-Lambert law for energy absorption inside the cornea. The eye was modeled as comprising five distinct homogeneous thermally isotropic regions. The numerical solution to the model was obtained and the computational results for the corneal temperature field were presented through the graphs and discussed. Singh et al. [12] introduced a validated 3D finite volume model of the human eye to study laser keratoplasty surgery. They considered the Pennes bio-heat equation for heat transfer and incorporated fluid convection in the model. The eye was treated as a region consisting of six homogeneous isotropic subdomains.

According to the review of literature, it is seen that the Pennes bio-heat equation is the most widely applied model for temperature distribution in living biological tissues. Fourier law assumes that heat flux and temperature change at any point appear at the same time instant. This implies that thermal signals propagate at an infinite speed [13]. Heat is always found to propagate at a finite speed. On the other hand, biological tissue has an inhomogeneous inner structure and found a phase lag time value. Mitra et al. [14] have shown experimental evidence of hyperbolic heat transfer in processed meat. They have used meat at different temperatures and brought them into contact suddenly and used thermocouples to measure the instantaneous temperature distributions. They have estimated a thermal phase lag time value to be around 16 sec. Kaminski [15] has also conducted some experiments with material with non-homogeneous inner structure and has found a phase lag time value of 20 sec. From the temperature distributions obtained, it can be inferred that parabolic and hyperbolic models produce similar results for smaller phase lag times, while they predict significantly different results when the lag times are large, suggesting that hyperbolic models are suitable to be used for biological systems with very complex internal structure. Gheitaghy et al. [16] proposed parabolic and hyperbolic models for heat transfer in the cornea under LTK surgery by considering the semi-infinite model of the cornea and solved the models using the mathematical analogy between thermal and electrical systems. The evaporation of heat at the anterior corneal surface was not considered. The computational results were presented by graphs and discussed.

In view of the importance and necessity of considering the thermal wave effects in bioheat transfer problems involving high heat flux incident on the tissue surface with a short duration, there is a need to formulate a hyperbolic model for the heat transfer in the corneal tissue subjected to short-pulsed Ho: YAG LTK irradiation. In the present study, a hyperbolic heat conduction model for the corneal temperature during LTK treatment to correct the refractive error in hyperopia is proposed by treating the cornea as a finite domain. Also, the evaporation of heat at the anterior corneal surface is incorporated into the model. The analytical solution to the model is derived using the finite cosine Fourier transform and the method of variation of parameters. The computational results for the temperature distribution and the temperature variation with time are presented by graphs and the influence of some model parameters on the distribution and variation are illustrated. Also, the degree of thermal damage in the cornea is evaluated and discussed.

2 Physical Model and Mathematical Formulation

The cornea subjected to short-pulsed laser irradiation at its anterior surface under LTK treatment is treated as a homogeneous finite domain of thickness L with constant thermal properties and thermally insulated bottom (endothelium surface) boundary. The Beer-Lambert Law, which experimentally asserts that the intensity of radiation drops exponentially with penetration depth, is used to describe the laser heat source.

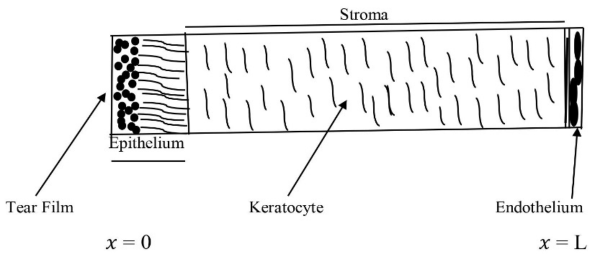

Fig. 1 Schematic diagram of the cornea modeled as a finite domain.

The heat transfer in living biological tissues is governed by Pennes model [17]:

$$\rho c \frac{\partial T(x,t)}{\partial t} = -\Delta \cdot q(x,t) + \\ + \rho_b c_b \omega_b (T_b - T(x,t)) + q_{mb} + Q(x,t), \quad (1)$$

where $\rho(\text{kg}\cdot\text{m}^{-3})$ is the tissue density, $c(\text{J}\cdot\text{kg}^{-1}\cdot{}^\circ\text{C}^{-1})$ is the tissue-specific heat, $q(\text{W}\cdot\text{m}^{-2})$ is the heat flux, $\rho_b(\text{kg}\cdot\text{m}^{-3})$ is the blood density, $c_b(\text{J}\cdot\text{kg}^{-1}\cdot{}^\circ\text{C}^{-1})$ is the blood specific heat, $\omega_b(\text{s}^{-1})$ is the volumetric blood perfusion rate per unit volume, $T_b({}^\circ\text{C})$ is the blood temperature, $T({}^\circ\text{C})$ is the tissue temperature, $q_{mb}(\text{W}\cdot\text{m}^{-3})$ is the heat generation due to metabolism heat and $Q(\text{W}\cdot\text{m}^{-3})$ is the heat generated by laser energy and x,t are the space and time coordinates, respectively. The laser heat source $Q(x,t)$ is described as follows:

$$Q(x,t) = (1-f)\cdot I(t)\cdot \mu e^{-\mu x},$$

where f is the Fresnel surface reflectance, $I(t)(\text{W}\cdot\text{m}^{-2})$ is the laser intensity and $\mu(\text{m}^{-1})$ is the absorption coefficient.

It could be overlooked due to the relatively small amount of metabolic heat compared to the heat generated by laser accidents [16]. Moreover, since corneal tissue is avascular, the blood perfusion rate at the targeted location is nearly zero. Therefore, $q_{mb} = 0, w_b = 0$. Now, Eq. (1) reduces to the following form:

$$\rho c\frac{\partial T(x,t)}{\partial t}=-\Delta\cdot q(x,t)+Q(x,t). \tag{2}$$

The Fourier law of heat conduction, which is a simple linear empirical relation between the heat flux vector and the temperature gradient is given by

$$q(x,t)=-k\frac{\partial T(x,t)}{\partial x}, \tag{3}$$

where $k(\mathrm{W\cdot m^{-1}\cdot {}^\circ C^{-1}})$ is the corneal tissue thermal conductivity.

The hyperbolic model is obtained by adding the parameter τ_q to time-variable t in $q(x,t)$ of Fourier law Eq. (4), is given by

$$q(x,t+\tau_q)=-k\frac{\partial T(x,t)}{\partial x}, \tag{4}$$

where τ_q is the relaxation time of heat flux. Taylor series expansion of first-order approximation of Eq. (4) with respect to time t is given by

$$q(x,t)+\tau_q\frac{\partial q(x,t)}{\partial t}=-k\frac{\partial T(x,t)}{\partial x}. \tag{5}$$

On solving Eqs. (2) and (5), we get a hyperbolic model of the bioheat transfer equation:

$$\tau_q\frac{\partial^2 T(x,t)}{\partial t^2}+\frac{\partial T(x,t)}{\partial t}=\alpha\frac{\partial^2 T(x,t)}{\partial x^2}+ +\frac{1}{\rho c}[Q(x,t)+\tau_q\frac{\partial Q(x,t)}{\partial t}], \tag{6}$$

where $\alpha=\frac{k}{\rho c}(\mathrm{cm^2\cdot sec^{-1}})$ the thermal diffusivity of the corneal tissue.

To solve the hyperbolic bioheat conduction Eq. (6) the following physically realistic and mathematically consistent conditions are prescribed.

Initial conditions:

$$T(x,t)=T_0,\ \text{at}\ t=0, \tag{7}$$

$$\frac{\partial T(x,t)}{\partial t}=0,\ \text{at}\ t=0. \tag{8}$$

Boundary conditions:

$$-k\frac{\partial T(x,t)}{\partial x}=h_0(T(x,t)-T_0)+ +\sigma\varepsilon(T^4(x,t)-T_0^4)+E, \tag{9}$$

at $x=0$,

$$\frac{\partial T(x,t)}{\partial x}=0,\ \text{at}\ x=L, \tag{10}$$

where $h_0(\mathrm{W\cdot m^{-2}\cdot K^{-1}})$ the convection coefficient, T_0 (°C) initial tissue temperature, $\sigma(\mathrm{W\cdot m^{-2}\cdot K^{-4}})$ the Stefan Boltzmann constant, ε the emissivity of the corneal surface, $E(\mathrm{W\cdot m^{-2}})$ the evaporative heat loss.

The heat loss at the anterior surface of the cornea is caused by convection, emission, and evaporation, as per the boundary condition (10). The posterior surface of the cornea is thermally insulated, according to boundary condition (11).

To non-dimensionalize the governing equation, the following scheme is introduced:

$$X=\frac{\omega x}{2\alpha},\tau=\frac{t}{2\tau_q},\psi=\frac{Q\tau_q}{\rho c T_0},\theta=\frac{T}{T_0}.$$

The dimensionless form of Eq. (7) is

$$\frac{\partial^2\theta}{\partial\tau^2}+2\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial X^2}+2\psi_0\left[2\eta(\tau)+\frac{\partial\eta(\tau)}{\partial\tau}\right], \tag{11}$$

where $\Psi(X,\tau)=\psi_0\eta(\tau)e^{-\beta X}$, $\psi_0=\frac{\beta I_r(1-f)}{2\omega\rho c T_0}$, $\beta=2\omega\tau_q\mu$ and $\eta(\tau)$ is the dimensionless rate of energy absorbed in the tissue. The laser heat source term is expressed in terms of arbitrary reference laser intensity I_r as $I(\tau)=I_r\,\eta(\tau)$.

The dimensionless forms of the initial and boundary conditions are

$$\theta(X,\tau)=1,\ \text{at}\ \tau=0, \tag{12}$$

$$\frac{\partial\theta(X,\tau)}{\partial\tau}=0,\ \text{at}\ \tau=0, \tag{13}$$

$$-B\frac{\partial\theta}{\partial X}=F(\theta-1)+H(\tilde\theta^2+1)(\tilde\theta+1)(\theta-1)+G, \tag{14}$$

at $X=0$,

$$\frac{\partial\theta}{\partial X}=0,\ \text{at}\ X=\frac{\omega L}{2\alpha}, \tag{15}$$

where $F=\frac{hT_0}{I_r},H=\frac{\sigma\varepsilon T_0^4}{I_r},G=\frac{E}{I_r},B=\frac{\omega\rho c T_0}{2I_r}.$

3 Solution to the Mathematical Model

Taking the Finite cosine transform of Eq. (11) and using boundary conditions (14) and (15) yields:

$$\frac{\partial^2\overline{\theta(n,\tau)}}{\partial\tau^2}+2\frac{\partial\overline{\theta(n,\tau)}}{\partial\tau}+\lambda_n^2\overline{\theta(n,\tau)}= =\frac{1}{B}\left[\begin{matrix}F(\theta(0,\tau)-1)+H(\tilde{\theta}^2+1)(\tilde{\theta}+1)\times\\ \times(\theta(0,\tau)-1)+G\end{matrix}\right]+ +\frac{2\psi_0\beta}{\lambda_n^2+\beta^2}\left[1-(-1)^n\exp(-\beta L)\right]\times \times\left[2\eta(\tau)+\frac{\partial\eta(\tau)}{\partial\tau}\right], \tag{16}$$

where $\lambda_n=\frac{n\pi}{L}$ ($n=0,\ 1,\ 2,\ 3...$ is the finite Fourier discrete variable) are the eigenvalues and $\overline{\theta(n,\tau)}=\int_0^L\theta(X,\tau)\cos(\lambda_n X)\mathrm{d}X$ is the finite cosine transformation of dimensionless temperature.

Moreover, the transformed initial conditions (12) and (13) are $\overline{\theta(n,0)}=0,\frac{\mathrm{d}\overline{\theta(n,0)}}{\mathrm{d}\tau}=0$.

The complementary solution of Eq. (16) is

$$\overline{\theta(n,\tau)}_c=C_1\exp(\sigma_n\tau)+C_2\exp(\delta_n\tau),$$

where C_1 and C_2 are constants to be determined, $\sigma_n=-1-\sqrt{1-\lambda_n^2}$ and $\delta_n=-1+\sqrt{1-\lambda_n^2}$.

Wronskian $W=(\delta_n-\sigma_n)\exp(\delta_n+\sigma_n)\tau=2\sqrt{1-\lambda_n^2}\exp(-2\tau)$.

Using the method of variation of parameters, the Particular solution of Eq. (16) is

$$\overline{\theta(n,\tau)}_p= =\frac{1}{\sqrt{1-\lambda_n^2}}\left[\begin{matrix}\int_0^\tau\left\{\begin{matrix}\frac{1}{B}(F(\theta(0,r)-1+H(\tilde{\theta}^2+1)\times\\ \times(\tilde{\theta}+1)(\theta(0,r)-1+G)+\\ +\frac{2\psi_0\beta}{\lambda_n^2+\beta^2}\left[1-(-1)^n\exp(-\beta L)\right]\times\\ \times\left(2\eta(\tau)+\frac{\partial\eta(\tau)}{\partial\tau}\right)\end{matrix}\right\}\times\\ \times\exp(-(\tau-r))\sinh\left(\sqrt{1-\lambda_n^2}(\tau-r)\right)\mathrm{d}r\end{matrix}\right].$$

Using the transformed initial conditions of Eqs. (12) and (13) the arbitrary constants C_1 and C_2 can be determined as:

$$C_1=\frac{1}{2(1-\lambda_n^2)}\left.\frac{\mathrm{d}S}{\mathrm{d}\tau}\right|_{\tau=0},C_2=-\frac{1}{2(1-\lambda_n^2)}\left.\frac{\mathrm{d}S}{\mathrm{d}\tau}\right|_{\tau=0},$$

where

$$S=\left[\begin{matrix}\int_0^\tau\left\{\begin{matrix}\frac{1}{B}(F(\theta(0,r)-1)+H(\tilde{\theta}^2+1)\times\\ \times(\theta+1)(\theta(0,r)-1+G)+\\ +\frac{2\psi_0\beta}{\lambda_n^2+\beta^2}\left[1-(-1)^n\exp(-\beta L)\right]\times\\ \times\left(2\eta(\tau)+\frac{\partial\eta(r)}{\partial\tau}\right)\end{matrix}\right\}\times\\ \times\exp(-(\tau-r))\sinh\left(\sqrt{1-\lambda_n^2}(\tau-r)\right)\mathrm{d}r\end{matrix}\right].$$

The general solution of Eq. (16) is

$$\overline{\theta(n,\tau)}=\frac{1}{2(1-\lambda_n^2)}\left.\frac{\mathrm{d}S}{\mathrm{d}\tau}\right|_{\tau=0}\exp(\sigma_n\tau)- -\frac{1}{2(1-\lambda_n^2)}\left.\frac{\mathrm{d}S}{\mathrm{d}\tau}\right|_{\tau=0}\exp(\delta_n\tau)+\frac{S}{\sqrt{1-\lambda_n^2}}. \tag{17}$$

Taking the inverse finite Fourier transform of Eq. (17) using Eq. (18) to get the temperature distribution during LTK surgery

$$\theta(X,\tau)=\frac{\overline{\theta(0,\tau)}}{L}+\frac{2}{L}\sum_{n=1}^{\infty}\overline{\theta(n,\tau)}\cos(\lambda_n X). \tag{18}$$

3.1 Thermal Damage

The smooth epithelial surface of the cornea blocks the passage of foreign materials such as dust, water and bacteria in the eye and absorbs oxygen and nutrients from the tears to supply to the rest of the cornea. To keep corneal hydration at its natural physiologic state, the endothelium actively pumps fluids into the anterior chamber. Maintaining adequate hydration is necessary for corneal opacity. During the LTK treatment procedure for hyperopia, the corneal temperature may rise to levels that cause significant epithelium surface vaporisation and endothelium thermal damage. The monitoring of temperature rise and maintaining optimal temperature during the treatment is required to avoid epithelium vaporisation and endothelium thermal damage. A numerical measure of the degree of damage in the corneal tissue may be a useful measure in the understanding and monitoring of corneal temperature.

According to Fyodorov et al. [18], the minimum corneal temperature should be 55 °C, Haw and Manche [19], the maximum temperature allowed is 75 °C, and Asbell et al. [20], the best temperature for optimal cornea collagen shrinking is 65 °C. Additionally, Aksan and McGrath [21] demonstrated the need of avoiding high temperatures and extensive denaturation to prevent tissue stiffness. As a result, the safe shrinking temperature range is 55–75 °C. Shrinkage occurs primarily during laser pulses and ceases immediately after the cooling phase between laser pulses begins. When the temperature of the corneal surface (epithelium) rises above 100 °C during treatment, thermal damage begins, indicating that the epithelium is at risk of superficial vaporisation. The following well-known Arrhenius equation has been widely utilised to assess thermal damage in biological tissue. The damage integral given by

$$\Omega(\tau,\theta)=\int_0^{\tau} A\exp(-\frac{E_a}{R\theta})\mathrm{d}\tau,$$

where $A(\sec^{-1})$ is a rate constant, $E_a(\mathrm{J}\cdot\mathrm{mol}^{-1})$ activation energy and $R(\mathrm{J}\cdot\mathrm{mol}^{-1}\cdot\mathrm{K}^{-1})$ the universal gas constant. The upper limit of the integral is usually taken to be the time at which the tissue returns to its initial temperature.

4 Results and Discussion

The computational results for the temperature profile and temperature variation with time in corneal tissue irradiated by Ho: YAG laser under LTK treatment are obtained using the values of typical parameters listed in the table below and displayed as graphs.

The variation of the degree of damage in the corneal tissue with time caused by laser irradiation during LTK surgery is portrayed in Fig. 2. It is evident from the graph that the highest degree of damage ($\Omega = 0.18$) is detected at $\tau = 0.7$ in the central stroma of the cornea, which is consistent with the literature [22]. In view of the observations [22, 23] that $\Omega = 0.1$ corresponds to the commencement of collagen denaturation and $\Omega > 1$ implies irreversible thermal damage in the tissue, it is discovered that collagen denaturation occurs during the time interval [0.36, 0.7] of the treatment. As a result, proper measures should be taken during LTK surgery to avoid the severity of thermal damage. By restricting the duration of laser heat, $\tau < 0.36$, the collagen denaturation in the corneal stroma may be avoided.

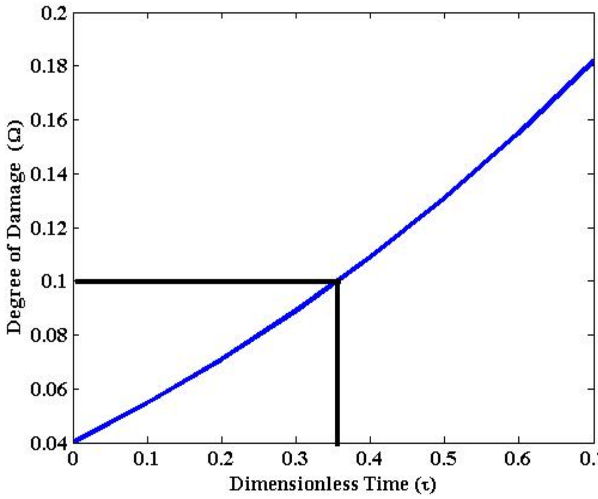

Fig. 2 Time-dependent degree of damage in the corneal tissue during Ho:YAG LTK.

Table 1 Parameters used in Computations.

Parameter	Symbol	Magnitude	Dimension
Corneal density	P	1062	$\mathrm{kg}\cdot\mathrm{m}^{-3}$
Corneal thermal conductivity	k	0.556	$\mathrm{W}\cdot\mathrm{m}^{-1}\cdot{}^{\circ}\mathrm{C}^{-1}$
Fresnel surface reflectance	f	2.4%	–
Absorption coefficient	M	2000	m^{-1}
Thermal diffusivity	α	0.001452	$\mathrm{cm}^2\cdot\sec^{-1}$
Emissivity of cornea	ε	0.975	–
Universal gas constant	R	8.32	$\mathrm{J}\cdot\mathrm{mol}^{-1}\cdot\mathrm{K}^{-1}$
Activation energy	E_a	402.688	$\mathrm{J}\cdot\mathrm{mol}^{-1}$
Rate constant	A	$3.13\cdot10^{61}$	$\sec^{-1}$
Stefan Boltzmann constant	σ	$5.67\cdot10^{-8}$	$\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-4}$
Wavelength	Λ	2.1	μm
Specific heat	C	3830	$\mathrm{J}\cdot kg^{-1}\cdot\mathrm{K}^{-1}$
Initial temperature	T_0	35	℃
Evaporative heat loss	E	40	$\mathrm{W}\cdot\mathrm{m}^{-2}$
Phase lag in heat flux	τ_q	1	sec
Thickness of cornea	L	0.55	Mm
Reference laser intensity	I_r	$2\cdot10^4$	$\mathrm{W}\cdot\mathrm{m}^{-2}$
Convection coefficient	H	20	$\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}$
Pulse duration	t_i	200	μ · sec

A comparison of the corneal temperature variations with time predicted by the parabolic model and the hyperbolic model is illustrated in Fig. 3. The temperature response predicted by Fourier's model (parabolic model) decreases with time rapidly, whereas the temperature predicted by the hyperbolic model decreases with time slowly. Most of the time, the temperature predicted by the hyperbolic model is higher than that predicted by the parabolic model. Similar comparisons of the theoretical results have been presented by H. Moosavi et al. [24] and Gheitaghy et al. [16].

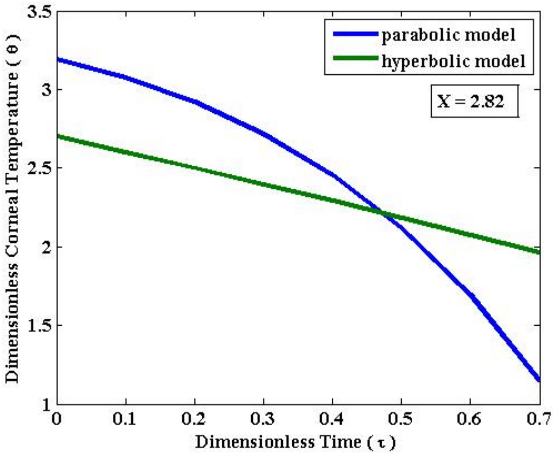

Fig. 3 A comparison of Fourier's model and the hyperbolic model anticipated dimensionless temperature variations over time during irradiation with a pulsed Ho: YAG laser.

The corneal temperature profiles at three different time durations have been shown in Fig. 4. It is observed from the curves that the temperature decreases along the length of the cornea. This behaviour occurs because the heat absorbed by the cornea decreases with the depth of the cornea. If the value of time duration is increased, the temperature rises throughout the cornea.

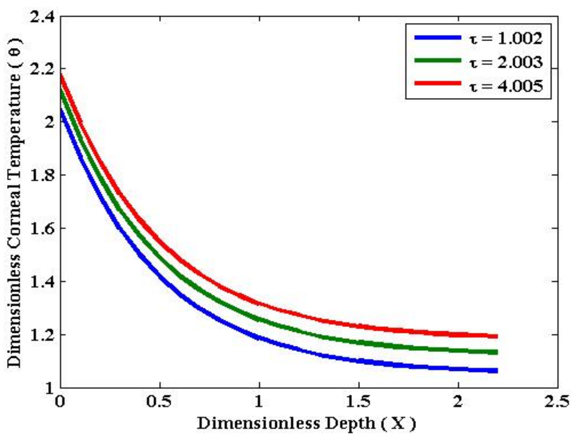

Fig. 4 Corneal temperature distribution at different times during pulsed Ho:YAG laser irradiation.

Fig. 5 depicts the influence of the convection coefficient on the temperature variations with time at the anterior corneal surface. The convection coefficient appears to have an insignificant impact on the temperature changes. During the time under discussion, the temperature at the anterior surface decreases with time. A similar result was predicted by the model of A. M. Gheitaghy et al. [16].

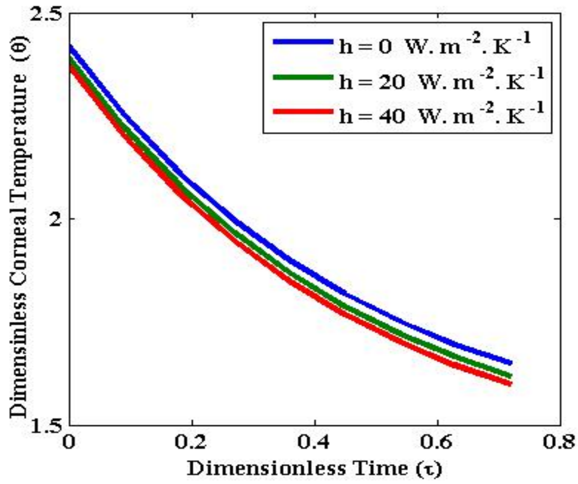

Fig. 5 The effect of the convection coefficient on the temperature variation with time predicted by the hyperbolic model.

Fig. 6 depicts the effect of the relaxation time on the dimensionless corneal temperature profile. It is observed from the figure that with an increase in the relaxation time of heat flux, the temperature rises and that the wavefront is more observable in the case of larger τ_q. Since τ_q is normally interpreted as the non-zero time that accounts for the effect of "thermal inertia", τ_q is responsible for the delay in establishing heat flux and associated conduction through the medium. This agrees qualitatively with the theoretical results of A. M. Gheitaghy et al. [16].

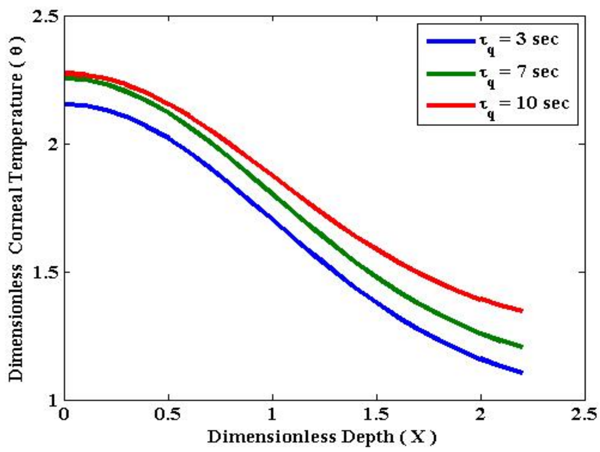

Fig. 6 The Temperature profile at various τ_q values in the corneal tissue at $\tau = 0$.

5 Conclusion

In this study, the hyperbolic heat conduction model is used to describe temperature changes in the human cornea exposed to short-pulsed Ho:YAG laser heating during LTK surgery, as well as to investigate the effect of phase-lags in the heat-flux on the corneal temperature distribution and temperature variation over time. It is observed that the temperature response anticipated by Fourier's model (Parabolic model) falls rapidly with time, whereas the temperature reduction predicted by the hyperbolic slows down with time. The temperature of the cornea rises as the heat flux phase-lag increases. It is necessary to verify the accuracy of the current findings by comparing the analytical results to the experimental data. Once, if the model is validated in near future, it may be utilised to improve the design of laser surgery for hyperopia treatment.

Disclosures

All authors declare that there is no conflict of interests in this paper.

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