Journal of Advances in Mathematics and Computer Science

Volume 37, Issue 12, Page 124-142, 2022; Article no.JAMCS.94946

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

Finite-time Sliding Mode Control for Interval Type-II Markov Jump Systems with Partially Known Transition Probabilities under Dynamic Event-triggered Scheme

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2022/v37i121734

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/94946

Received: 20/10/2022

Accepted: 26/12/2022

Published: 27/12/2022

Original Research Article

Abstract

The issue of finite-time event-triggered sliding mode control (SMC) is investigated for a class of interval type-II fuzzy Markov jump systems with partially known transition probabilities. Firstly, for the sake of saving network resources, a dynamic event-triggered scheme (DETS) is proposed to determine whether to transmit the signal or not. Then, a feasible SMC law is developed that makes the state trajectory of the system reach

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the specified sliding surface in finite-time. Thereafter, by means of the time partition strategy, sufficient conditions for the system to be bounded in finite-time during the arrival and sliding stages are derived. Additionally, the controller gains are computed by utilizing the linear matrix inequality (LMI) toolbox. Lastly, the advantages of the SMC strategy are verified by simulation products.

Keywords: Finite-time sliding mode control; dynamic event-triggered scheme (DETS); interval type-II Markov jump systems.

1 Introduction

It is well known that the Takagi-Sugeno (T-S) fuzzy system is an excellent tool for the analysis and synthesis of complex nonlinear plants[1]-[5]. With the T-S fuzzy model, the primitive nonlinear system can be well expressed by a "mixture" of some local linear subsystems. Consequently, analysis of the stability of linear systems can be prolonged to the nonlinear case by the T-S fuzzy modeling approach.

Markovian jump systems (MJSs) as a special class of switched system which consist of an indexed family of subsystems and a set of Markovian chain, have obtained increasing attention. Different from the conventional controller, the control design of MJSs is under the idea of switching to improve the performance of closed-loop system. During the past few years, MJSs have been applied to various fields of science and engineering and a great number of research works have been reported [6]-[11] and references therein. However, most of the researcher have only considered the transition probabilities of MJSs are fully known and various results have been investigated in [12][13]. In practical point of view, the transition probabilities of MJSs are not fully known, due to the presence of channel delays and packet dropout in digital control system, it may be very costly to achieve full known transition probabilities [14][15]. Hence, it is important and necessary to further investigate more general MJSs together with partially known transition probabilities.

The event-triggered mechanism, as an effective way to reduce communication burdens, has been widely applied, whose main idea is that the control/measurement signal can access the network only at the instant when the triggering condition is satisfied [16]. Compared with the static event-triggered scheme(SETS), the dynamic event-triggered scheme (DETS) has received increasing attention in recent years [17]-[20]. The key feature of the dynamic event-triggered (DET) strategy is that an additional internal dynamical variable is introduced to adjust the event-triggered condition adaptively. In [20], the DET method was also integrated into the SMC scheme to stabilize a slow-sampling singularly perturbed system. Nevertheless, the application of DETS to interval type-II fuzzy MJSs with partially known transition rates, there are not many relevant research results. This is the main motivation of this paper.

Furthermore, sliding mode control (SMC), a discontinuous nonlinear control method, has been extensively studied for several decades due to its simplicity and robustness. The main idea behind SMC is to drive the system states onto a prescribed sliding surface in finite-time and to maintain the trajectory on it for all subsequent time [21]-[24]. In [25]-[27] , it is highlighted that Lyapunov stability takes into account the fact that the system's trajectory achieves a balance point within an infinitesimal time lapse. These articles only focus on the infinite-time reachability, but in engineering applications, there is an urgent need to actuate the trajectory of the system onto a specified sliding surface within a limited timespan. Consequently, a growing interest in the field of finite-time stability (FTS) and finite-time boundedness (FTB) conceptions has been observed in the recent couple of years, following the increase in efficiency of practical systems. In [28], FTB and reachability of MJSs with timelags are discussed. Besides, scholars have also extended the finite-time theory to fuzzy systems [29], multi-agent system systems [30], randomly switched systems [31] and so on.

In consideration of the above discussion, the issue of finite-time DET SMC for interval type-II fuzzy MJSs with partially known transition probabilities, which has greatly aroused our attention. The key contributions to this article are outlined below:

1. The DET SMC problem of interval type-II fuzzy MJSs with partially known transition rates has been considered, and the minimum inter-event time has been given to avoid Zeno phenomenon.
2. In view of the effect of DETS, the concept of FTB is introduced to cope with the mismatched membership functions(MFs) and the exterior interferences. The trajectories of the interval type-II fuzzy MJSs with partially known transition rates are not only ensured to arrive at the specified sliding surface in finite-time by utilizing the designed SMC strategy, but also sufficient conditions for the system to be bounded in finite-time.
3. An example is given to demonstrate that the proposed DETS is more effective than the traditional SETS.

Notations: The symbols applicable in the present paper are generic. $\alpha_{\max}(C)$ means the maximum eigenvalue of matrix C, with $\|C\|$ represents the Euclidean norm of C. $E\{\cdot\}$ indicates the mathematical expectation. $C > 0$ represents that C is symmetric positive-definite. " $*$ " represents the symmetric block for a symmetric matrix. $sgn(c)$ represents the sign function that equals 1 when $c > 0$, equals 0 when c = 0. $sym(C) = C + C^T$. If there are no special instructions, the matrices are considered to have appropriate dimensions.

2 Problem Formulation and Preliminaries

2.1 System description

With respect to the probabilistic space $(\uplus, \odot, \mathrm{Pr})$, the nonlinear MJSs with external disturbance are constructed by the T-S fuzzy model presented below:
Plant rule i: IF $\beth_1(x(t))$ is $\eth_1^i$,..., and $\beth_p(x(t))$ is $\eth_p^i$, THEN

$$\begin{cases} \dot{x}(t) = A_i(\ddagger(t))x(t) + B_i(\ddagger(t))u(t) + D_i(\ddagger(t))w(t) \\ y(t) = C(\ddagger(t))x(t) \end{cases} \tag{1}$$

where $\beth_\jmath(x(t))$ is the premise variables, $\eth_\jmath^i$ represents the fuzzy aggregations, $i \in \digamma = \{1, 2, \ldots, v\}$, $\jmath \in \mho = \{1, 2, \ldots, p\}$ and p is the number of premise variables and v is the number of fuzzy rules. $u(t)$ and $x(t)$ signify the control input and the system state, separately; $y(t)$ is the output vector; $w(t)$ is the exterior interference with a known upper bound $\bar{w}$ meeting $\|w(t)\| \leq \|\bar{w}\|$. The firing intensity is for the rule i specified as the below given set of intervals: $\eta_i(x(t)) = \left[\underline{\eta}_i(x(t)), \overline{\eta}_i(x(t))\right]$ with

$$\underline{\eta}_i(x(t)) = \prod_{\jmath=1}^{p} \underline{\mu}_{\eth_\jmath^i}\left(\daleth_\jmath(x(t))\right) > 0, \overline{\eta}_i(x(t)) = \prod_{\jmath=1}^{p} \overline{\mu}_{\eth_\jmath^i}\left(\daleth_\jmath(x(t))\right) > 0,$$

where $\underline{\mu}_{\eth_\jmath^i}\left(\beth_\jmath(x(t))\right)$ and $\overline{\mu}_{\eth_\jmath^i}\left(\beth_\jmath(x(t))\right) \in [0, 1)$ are the lower and upper grade of the MFs, satisfying $\underline{\mu}_{\eth_\jmath^i}\left(\beth_\jmath(x(t))\right) \leq \overline{\mu}_{\eth_\jmath^i}\left(\beth_\jmath(x(t))\right)$. $\{\ddagger(t)\}_{t \geq 0}$ is a Markovian process with its values within a finite set $\P \triangleq \{1, 2, \ldots, R\}$. Then, the transition rate matrix $\aleph \triangleq (\varphi_{\Im \pounds})$, $\Im, \pounds \in \P$ from mode $\Im$ at time t to mode $\pounds$ at time $t + \xi$ is determined by

$$Pr\left\{\ddagger_{t+\xi} = \pounds | \ddagger_t = \Im\right\} \triangleq \begin{cases} \varphi_{\Im \pounds}\xi + o(\xi), & \Im \neq \pounds; \\ 1 + \varphi_{\Im\Im}\xi + o(\xi), & \Im = \pounds; \end{cases}$$

where $\xi > 0$, $\lim\limits_{\xi \to 0}(o(\xi)/\xi) = 0$; $\varphi_{\Im \pounds} \geq 0$, $\Im \neq \pounds$, $\varphi_{\Im\Im} = -\sum\limits_{\pounds \neq \Im} \varphi_{\Im \pounds}, \forall \Im \in \P$.
In this article, it is hypothesized that the Markov jump parameters are partially known, meaning that the other parts of the elements of matrix $\aleph$ are unknown. By way of example, consider the following form of a system with three modes:

$$\aleph = \begin{bmatrix} \varphi_{11} & ? & ? \\ ? & \varphi_{22} & ? \\ ? & \varphi_{32} & ? \end{bmatrix}$$

where "?" represents the unknown elements. To streamline the symbols, define $\P = \P_{\lambda}^{\Im} + \P_{u\lambda}^{\Im}$, $\forall \Im \in \P$ with $\P_{\lambda}^{\Im} \triangleq \{\pounds | \varphi_{\Im\pounds} \text{ is known}\}$, $\P_{u\lambda}^{\Im} \triangleq \{\pounds | \varphi_{\Im\pounds} \text{ is unknown}\}$. If $\P_{\lambda}^{\Im} \neq \emptyset : \P_{\lambda}^{\Im} = \{\lambda_1^{\Im}, \ldots, \lambda_g^{\Im}\}, \forall 1 \leq g \leq R,$ where $\lambda_g^{\Im} \in N^+$ is the gth element in the $\Im th$ row of matrix $\aleph$.

To the whole $\ddagger(t) \triangleq \Im$, we define $A_{i\Im} \triangleq A_i(\ddagger(t)), B_{i\Im} \triangleq B_i(\ddagger(t)), C_{\Im} \triangleq C(\ddagger(t)), D_{i\Im} \triangleq D_i(\ddagger(t))$ to simplier the notations. Thus, the overall considered system can be formulated via the below T-S fuzzy model:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{v} \eta_i(x(t)) \left[A_{i\Im}x(t) + B_{i\Im}u(t) + D_{i\Im}w(t)\right] \\ y(t) = C_{\Im}x(t) \end{cases} \tag{2}$$

with

$$\begin{aligned} &\eta_i(x(t)) = \underline{\eta}_i(x(t))\underline{\ell}_i(x(t)) + \overline{\eta}_i(x(t))\overline{\ell}_i(x(t)) \\ &\underline{\ell}_i(x(t)), \overline{\ell}_i(x(t)) \in [0,1], \underline{\ell}_i(x(t)) + \overline{\ell}_i(x(t)) = 1, \sum_{i=1}^{v} \eta_i(x(t)) = 1 \end{aligned}$$

Where the the nonlinearity functions $\underline{\ell}_i(x(t))$ and $\overline{\ell}_i(x(t))$ enable uncertainty in parameters to be trapped, $A_{i\Im}, B_{i\Im}, C_{\Im}, D_{i\Im}$ are known matrices with appropriate dimensionalities.

2.2 Dynamic event-triggered scheme

In the paper, as presented in Fig.1, for the purpose of relieving the data transmission pressure, we introduce a dynamic event-triggered scheme(DETS) which is used to decide whether the system state $x(t)$ will be released into the controller . Define $e(t) = x(t) - x(t_k)$ as the difference between the current state $x(t)$ and the last transmitted state $x(t_k)$. The triggering instant t_{k+1} is decided by the following rule:

$$t_{k+1} = inf\left\{t | t > t_k, \frac{1}{\rho}\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t) \leq 0\right\} \tag{3}$$

where $\wp \in (0,1), \rho > 0$, N with M are selected parameters. In addition, the dynamic variable $\spadesuit(t)$ satisfies the following rule:

$$\dot{\spadesuit}(t) = -\kappa\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t) \tag{4}$$

where $\kappa > 0$ with $\spadesuit(0) = \spadesuit_0 \geq 0$.

Remark 2.1. *Different from the SETS in [8], the variable $\spadesuit(t)$, as the additional internal dynamic threshold, is introduced into the DET criterion (3), which can be adjusted adaptively along with the rule (4). Therefore, the presented DETS is more flexible in improving the transmission efficiency than the SETS.*

Next, our goal is to design a fuzzy SMC law to realize the finite-time boundedness(FTB) of interval type-II fuzzy MJSs (2) under DETS.

Lemma 2.1. *[28] Considering the real matrices e and f which have suitable dimensionality, with respect to any scalar $\sigma > 0$, the following inequality forms:*

$$e^T f + f^T e \leq e^T \sigma^{-1} e + f^T \sigma f \tag{5}$$

Definition 2.1. *[8] Fixed a time inteval $[0, \top]$, scalars $d_2 > d_1 > 0, \varsigma > 0$ and a matrix $\mathbb{R} > 0$, system(2) is FTB with respect to $\left(d_1, d_2, [0, \top], \mathbb{R}, \Omega_{[0,T],\varsigma}\right)$, if the following condition holds for any $t \in [0, \top]$*

$$\begin{cases} E\left\{x^T(0)\mathbb{R}x(0)\right\} \leq d_1 \\ \mathbb{R} \triangleq \int_0^{\top} w^T(t)w(t) < \varsigma \end{cases} \Rightarrow E\left\{x^T(t)\mathbb{R}x(t)\right\} \leq d_2 \tag{6}$$

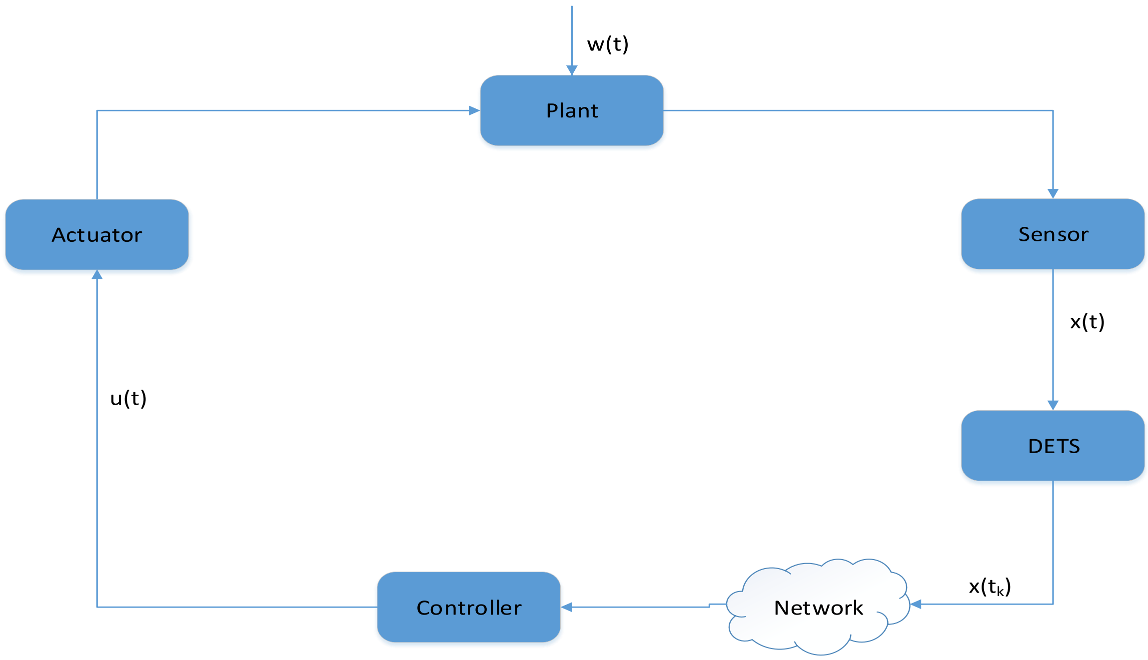

Fig.1. The system structure under DETS

3 Main Results

3.1 Sliding surface design

The designed sliding surface is as follows:

$$s(t) = \mathbb{J}x(t) \tag{7}$$

where $\mathbb{J} \triangleq \sum_{\Im=1}^{R} \nu_\Im B_\Im^T$, with scalars $\nu_\Im (\Im \in \P)$ are supposed to be selected such that $\mathbb{J}B_\Im$ is nonsingular for any $\Im \in \P$.

Plant Rule j : If $\flat_1(x(t))$ is Z_1^i, $\flat_2(x(t))$ is Z_2^i, ..., and $\flat_p(x(t))$ is Z_p^i, then the ambiguous sliding mode controller is designed below:

$$u(t) = \sum_{i=1}^{v} h_i(x(t_k))[K_{i\Im}x(t_k) - \gamma(t)sgn(s(t))] \tag{8}$$

where $\sum_{i=1}^{v} h_i(x(t_k)) = 1, h_i(x(t_k)) \in [0,1], i \in \digamma$, $\gamma(t) = \tau + \phi \|w(t)\| + \beta \|x(t)\| + \|K_{i\Im}x(t_k)\|$
with $\phi \triangleq \max_{\Im\in\P} \{\phi_\Im\}, \phi_\Im \triangleq \left\|(\mathbb{J}B_{i\Im})^{-1}\mathbb{J}D_{i\Im}\right\|, \beta \triangleq \max_{\Im\in\P} \{\beta_\Im\}, \beta_\Im \triangleq \frac{1}{2}\|\Pi_\Im\mathbb{J}\| + \left\|(\mathbb{J}B_{i\Im})^{-1}\mathbb{J}A_{i\Im}\right\|$,
$\Pi_\Im \triangleq \sum_{\pounds=1}^{R} \varphi_{\Im\pounds}(\mathbb{J}B_{i\Im})^{-1}$, $\tau > 0$ is a small scalar.

Remark 3.1. *In the previous work, the sliding surface therein is designed as a mode-dependent one. Nevertheless, due to the existence of the sign function, it is inevitable that the trajectory of the system raises the chattering phenomenon when reaching the sliding surface. Therefore, if the mode-dependent sliding surface is chosen, the reachability of the sliding surface might not be guaranteed all the time, and the trajectory of the system can not do sliding motions along with the sliding surface strictly. Furthermore, the frequent switching of the sliding surface and the chattering phenomenon of the system trajectory may degrade the performance of the controller. Thus, we adopt a mode-independent sliding surface to avoid the above defects in this article.*

3.2 Finite-time reachability analysis

In this part, within a given limited time $[0, \top]$, the SMC law $u(t)$ is designed to compel the trajectories of the systems into the sliding surface $s(t) = 0$ in a finite interval $[0, \top^*]$, and ensures that they stay on the sliding surface for the rest of time $[\top^*, \top]$.

Theorem 3.1. *For the interval type-II fuzzy MJSs (2), the SMC law (8) enables trajectories of the system to be compelled into the sliding surface $s(t) = 0$ in the limited time $[0, \top^*]$ $(\top^* < \top)$ and stay on there in $[\top^*, \top]$ in mean square sense, where τ in the SMC law (8) satisfies*

$$\tau \geq \frac{\max\limits_{\Im \in ¶}\{\alpha_{max}(\mathbb{J}B_{i\Im})^{-1}\}}{\top} \left\| \mathbb{J}x(0) \right\| \tag{9}$$

Proof: Consider the Lyapunov function with respect to any $t \in [0, \top]$

$$\mathbb{V}_1(s(t), \Im, t) = \frac{1}{2} s^T(t)(\mathbb{J}B_{i\Im})^{-1} s(t) \tag{10}$$

For the convenience of writing, let $\mathbb{V}_1(s(t), \Im, t) \triangleq \mathbb{V}_1(t)$ whose infinitesimal operator is obtained:

$$\begin{aligned}\Gamma V_1(t) =& \frac{1}{2} s^T(t)\Pi_{\Im} s(t) + s^T(t)(\mathbb{J}B_{i\Im}^{-1})\{\sum_{i=1}^{v} \eta_i(x(t))[\mathbb{J}A_{i\Im}x(t) + \mathbb{J}D_{i\Im}w(t) + \sum_{\pounds=1}^{R} \varphi_{\Im\pounds}(\mathbb{J}B_{i\Im}) \\ & (K_{i\Im}x(t_k) - \gamma(t)sgn(s(t)))]\} \\ \leq & -\tau \left\| s(t) \right\| \\ \leq & -\frac{\tau}{\varrho}\sqrt{V_1(t)}\end{aligned} \tag{11}$$

where $\varrho \triangleq \sqrt{\frac{\max\limits_{\Im \in ¶}\{\alpha_{max}(\mathbb{J}B_{i\Im})^{-1}\}}{2}}$

Integrating (11) from 0 to t with $t \in [0, \top^*]$, one gets that:

$$2\mathbb{V}_1^{\frac{1}{2}}(\top^*) - 2\mathbb{V}_1^{\frac{1}{2}}(0) \leq -\frac{\tau}{\varrho}\top^* \tag{12}$$

from which we known that there is $\mathbb{V}_1(\top^*) = 0(s(t) = 0$ for $t \geq \top^*)$. Thus, by virtue of (12), we can get that:

$$\top^* \leq \frac{2\varrho}{\tau}\sqrt{\mathbb{V}_1(0)} \tag{13}$$

In conjunction with (7) and (10), we can derive that $\mathbb{V}_1(0) \leq \frac{1}{2} \max\limits_{\Im \in ¶}\{\alpha_{max}(\mathbb{J}B_{i\Im})^{-1}\} \left\| s(0) \right\|^2$ and $\left\| s(0) \right\| = \left\| \mathbb{J}x(0) \right\|$, then one derives that:

$$\top^* \leq \frac{\max\limits_{\Im \in ¶}\{\alpha_{max}(\mathbb{J}B_{i\Im})^{-1}\}}{\tau} \left\| \mathbb{J}x(0) \right\| \tag{14}$$

In combination with (9), we get $\top^* \leq \top$.

Consequently, in a limited time $[0, \top]$, the trajectories of the interval type-II fuzzy MJSs (2) is to be compelled into the sliding surface (7) within limited time $\top^*(\top^* < \top)$, and maintained there during the rest of time $[\top^*, \top]$, the proof is thus accomplished.

Remark 3.2. *It is clear from the certification procedure described above that the parameter τ in the SMC (8) is important and that it has the power to determine the arrival time $\top^*$. According to (14), the time to the arrival stage decreases as τ increases in value.*

Remark 3.3. *The analysis of arrival stage $[0, \top^*]$ and sliding mode movement stage $[\top^*, \top]$ in chunks makes finite-time sliding mode control distinct. In accordance with **Theorem 3.1**, the trajectories of the system will be driven to the sliding surface $s(t) = 0$ in finite-time $\top^*$. The next step is to demonstrate that the closed-loop system(CLS), along with the sliding surface in $[0, \top]$, is mean-square finite-time bounded.*

3.3 Finite-time boundedness in $[0, \top^*]$

During the subsection, the trajectories of the system are at the external side of the sliding surface within the arrival phase $[0, \top^*]$, which means $s(t) \neq 0$. By incorporating (2) and (8), the CLS can be reprofiled as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j [\breve{A}_{i\Im} x(t) - B_{i\Im} K_{i\Im} e(t) - B_{i\Im} \bar{\gamma}(t) + D_{i\Im} w(t)] \\ y(t) = C_{\Im} x(t) \end{cases} \tag{15}$$

where $\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \triangleq \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i(x(t)) h_j(x(t_k)), \breve{A}_{i\Im} \triangleq A_{i\Im} + B_{i\Im} K_{i\Im}, \bar{\gamma}(t) \triangleq \gamma(t)\,\mathrm{sgn}(s(t))$. And we can get:

$$\begin{aligned} \bar{\gamma}^T(t)\bar{\gamma}(t) =& [\tau + \phi\,\|w(t)\| + \beta\,\|x(t)\| + \|K_{i\Im} x(t_k)\|]^T [\tau + \phi\,\|w(t)\| + \beta\,\|x(t)\| + \|K_{i\Im} x(t_k)\|] \\ \leq& 4\tau^2 + 4\phi^2 w^T(t) w(t) + 4\beta^2 x^T(t) x(t) + 4x^T(t_k) K_{i\Im}^T K_{i\Im} x(t_k) \end{aligned} \tag{16}$$

Then, the FTB issue of the system (15) is to be investigated within the time $[0, \top^*]$.

Theorem 3.2. *With respect to the SMC law (8), given parameters $d_2 > d_1 > 0, \lambda > 0, \varsigma > 0, \tau > 0, \kappa > 0, \rho > 0, \wp > 0, N, M$ and matrix $\mathbb{R} > 0$ are predetermined. If there are scalars $d^* > 0, \top^* > 0$, matrices $\mathbb{P}_{i\Im} > 0, \mathbb{Z} > 0, \mathbb{Q} > 0$ and symmetric matrices $L_{\Im}$, such that the following conidtions hold for all $i, j \in \digamma, \Im \in \P$*

$$\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j W_{ij} < 0 \tag{17}$$

$$\mathbb{P}_{i\pounds} - L_{\Im} \leq 0, \pounds \in \P_{u\lambda}^{\Im}, \pounds \neq \Im \tag{18}$$

$$\mathbb{P}_{i\pounds} - L_{\Im} \geq 0, \pounds \in \P_{u\lambda}^{\Im}, \pounds = \Im \tag{19}$$

$$4\lambda\beta^2 I < e^{-\lambda\top^*}\mathbb{Z} \tag{20}$$

$$4\lambda K_{i\Im}^T K_{i\Im} < e^{-\lambda\top^*}\mathbb{Q} \tag{21}$$

$$d_1 < d^* < d_2 \tag{22}$$

$$\frac{\bar{\eth}_{\mathbb{P}_{i\Im}} d_1 + \spadesuit_0 + 4\tau^2\lambda\top^* + 4\lambda\varsigma\phi^2 + \lambda\varsigma}{\underline{\eth}_{\mathbb{P}_{i\Im}}} < e^{-\lambda\top^*} d^* \tag{23}$$

where

$$W_{ij} = \begin{bmatrix} W_{11} & W_{12} & I \\ * & W_{22} & 0 \\ * & * & -\mathbb{Z}^{-1} \end{bmatrix}$$

$$W_{11} = \sum_{\pounds \in \P_{\lambda}^{\Im}} \varphi_{\Im\pounds}(\mathbb{P}_{i\pounds} - L_{\Im}) + \mathbb{P}_{i\Im}\tilde{A}_{i\Im} + \tilde{A}_{i\Im}^T \mathbb{P}_{i\Im} + \mathbb{Q} + (1 + k_1)\wp N - \lambda\mathbb{P}_{i\Im},$$

$W_{12} = \left[\mathbb{P}_{i\Im} D_{i\Im} \quad -\mathbb{P}_{i\Im} \sum_{\pounds=1}^{R} \varphi_{\Im\pounds} B_{i\Im} K_{i\pounds} - \mathbb{Q} \quad -\mathbb{P}_{i\Im} B_{i\Im} \quad 0\right],$
$W_{22} = -diag\left[\lambda I, (1+k_1)M - \mathbb{Q}, \lambda I, \kappa - \frac{k_1}{\rho} + \lambda\right]$
$\tilde{A}_{i\Im} \triangleq A_{i\Im} + \sum_{\pounds=1}^{R} \varphi_{\Im\pounds} B_{i\Im} K_{i\pounds}$
$\underline{\eth}_{\mathbb{P}_{i\Im}} \triangleq \min_{s\in\P} \left(\alpha_{min}\left(\mathbb{R}^{-\frac{1}{2}} \mathbb{P}_{i\Im} \mathbb{R}^{\frac{1}{2}}\right)\right),$
$\overline{\eth}_{\mathbb{P}_{i\Im}} \triangleq \max_{s\in\P} \left(\alpha_{max}\left(\mathbb{R}^{-\frac{1}{2}} \mathbb{P}_{i\Im} \mathbb{R}^{\frac{1}{2}}\right)\right)$

then, the system (15) is FTB about $\left(d_1, d^*, [0, \top^*], \mathbb{R}, \Omega_{[0,\top^*],\varsigma}\right)$.

Proof: Note that the DET condition (3) implies for any $t \in [t_k, t_{k+1}]$

$$\frac{1}{\rho}\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t) > 0 \tag{24}$$

By virtue of the dynamic equation(4),we get

$$\dot{\spadesuit}(t) > -\kappa\spadesuit(t) - \frac{1}{\rho}\spadesuit(t) = -(\kappa + \frac{1}{\rho})\spadesuit(t) \tag{25}$$

Then, integrating from 0 to t, yields

$$\spadesuit(t) > \spadesuit_0 e^{-(\kappa+\frac{1}{\rho})t} \tag{26}$$

For $\spadesuit_0 > 0$, we get $\spadesuit(t) > 0$ for $t \in [0, \infty)$.

Select the Lyapunov function:

$$\mathbb{V}_2(x(t), \Im, t, \spadesuit(t)) = x^T(t)\mathbb{P}_{i\Im}x(t) + \int_0^t x^T(\xi)\mathbb{Z}x(\xi)d\xi + \int_0^t x^T(t_k)\mathbb{Q}x(t_k)dt + \spadesuit(t) \tag{27}$$

Let $\mathbb{V}_2(t) \triangleq \mathbb{V}_2(x(t), \Im, t, \spadesuit(t))$, the weak infinitesimal generator of $\mathbb{V}_2(t)$ is given:

$$\begin{aligned}\Gamma\mathbb{V}_2(t) = {} & x^T(t) \sum_{\pounds=1}^{R} \varphi_{\Im\pounds}\mathbb{P}_{i\pounds}x(t) + sym\{x^T(t)\mathbb{P}_{i\Im} \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j[\tilde{A}_{i\Im}x(t) - B_{i\Im}\bar{\gamma}(t) + D_{i\Im}w(t) \\ & - \sum_{\pounds=1}^{R} \varphi_{\Im\pounds}B_{i\Im}K_{i\pounds}e(t)]\} + x^T(t)\mathbb{Z}x(t) + x^T(t)\mathbb{Q}x(t) + e^T(t)\mathbb{Q}e(t) - 2x^T(t)\mathbb{Q}e(t) \\ & -\kappa\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t)\end{aligned}$$

Due to $\sum_{\pounds=1}^{R} \varphi_{\Im\pounds}L_{\Im} = 0$ for arbitrary symmetric matrices $L_{\Im}$, so $\Gamma\mathbb{V}_2(t)$ can be rewritten as

$$\begin{aligned}\Gamma\mathbb{V}_2(t) = {} & x^T(t)[\sum_{\pounds=1}^{R} \varphi_{\Im\pounds}\mathbb{P}_{i\pounds} - \sum_{\pounds=1}^{R} \varphi_{\Im\pounds}L_{\Im}]x(t) + sym\{x^T(t)\mathbb{P}_{i\Im}\sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j[\tilde{A}_{i\Im}x(t) - B_{i\Im}\bar{\gamma}(t) \\ & + D_{i\Im}w(t) - \sum_{\pounds=1}^{R}\varphi_{\Im\pounds}B_{i\Im}K_{i\pounds}e(t)]\} + x^T(t)\mathbb{Z}x(t) + x^T(t)\mathbb{Q}x(t) + e^T(t)\mathbb{Q}e(t) \\ & - 2x^T(t)\mathbb{Q}e(t) - \kappa\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t) \\ = {} & x^T[\sum_{\pounds\in\P_{\lambda}^{\Im}} \varphi_{\Im\pounds}(\mathbb{P}_{i\pounds} - L_{\Im}) + \sum_{\pounds\in\P_{u\lambda}^{\Im}} \varphi_{\Im\pounds}(\mathbb{P}_{i\pounds} - L_{\Im}) + \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j(\mathbb{P}_{i\Im}\tilde{A}_{i\Im} + \tilde{A}_{i\Im}^T\mathbb{P}_{i\Im}) + \mathbb{Q} + \mathbb{Z} \\ & + \wp N]x(t) + sym\{x^T(t)\mathbb{P}_{i\Im}\sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j[-B_{i\Im}\bar{\gamma}(t) + D_{i\Im}w(t) - \sum_{\pounds=1}^{R}\varphi_{\Im\pounds}B_{i\Im}K_{i\pounds}e(t)]\} \\ & + e^T(t)\mathbb{Q}e(t) - 2x^T(t)\mathbb{Q}e(t) - \kappa\spadesuit(t) - e^T(t)Me(t)\end{aligned} \tag{28}$$

Select the auxiliary function
$H_1(\Im, t, \spadesuit(t)) = \Gamma\mathbb{V}_2(t) - \lambda\mathbb{V}_2(t) - \lambda\bar{\gamma}^T(t)\bar{\gamma}^T(t) - \lambda w^T(t)w(t)$
If the DET condition (3) is violated, for any scalar $k_1 > 0$, we can obtain from (28)

$$\begin{aligned} H_1(\Im, t, \spadesuit(t)) \leq & H_1(\Im, t, \spadesuit(t)) + k_1(\frac{1}{\rho}\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t)) \\ & + \int_0^t x^T(\xi)\mathbb{Z}x(\xi)d\xi + \int_0^t x^T(t_k)\mathbb{Q}x(t_k)dt \end{aligned} \tag{29}$$

Notice that $\varphi_{\Im\pounds} \geq 0$ for all $\Im \neq \pounds$ and $\varphi_{\Im\Im} = -\sum_{\pounds\neq\Im}\varphi_{\Im\pounds}$ for all $\Im \in \P_\lambda$, if $\Im \in \P_\lambda^\Im$ (the elements of the diagonal are known), by inequalities (17)-(18) and Schur complement, the following inequality holds:

$$\Gamma\mathbb{V}_2(t) < \lambda\mathbb{V}_2(t) + \lambda\bar{\gamma}^T(t)\gamma(t) + \lambda w^T(t)w(t) \tag{30}$$

If $\Im \in \P_{u\lambda}^\Im$ (the elements of the diagonal are unknown), according to inequalities (17)-(19) and Schur complement, the following (30) holds.

Employing $e^{-\lambda t}$ to multiply both sides of inequality (30), we can get that

$$\frac{e^{-\lambda t}d\mathbb{V}_2(t)}{dt} < \lambda e^{-\lambda t}\mathbb{V}_2(t) + \lambda e^{-\lambda t}w^T(t)w(t) + \lambda e^{-\lambda t}\bar{\gamma}^T(t)\bar{\gamma}(t) \tag{31}$$

For $t \in [0, \top^*]$, integrate the above inequality from 0 to t, which yields

$$\begin{aligned} & e^{-\lambda t}\mathbb{V}_2(t) < \mathbb{V}_2(0) + \lambda\int_0^t e^{-\lambda\xi}\bar{\gamma}^T(\xi)\bar{\gamma}(\xi)d\xi + \lambda\int_0^t e^{-\lambda\xi}w^T(\xi)w(\xi)d\xi \\ \leq & \bar{\eth}_{\mathbb{P}_{i\Im}}d_1 + \spadesuit_0 + (4\tau^2\lambda\top^* + 4\lambda\phi^2\varsigma + \lambda\varsigma) + \int_0^t x^T(\xi)(4\lambda\beta^2 I)x(\xi)d\xi + \int_0^t x^T(t_k)(4\lambda K_{i\Im}^T K_{i\Im})x(t_k)dt \end{aligned} \tag{32}$$

Besides, it is clear that

$$e^{-\lambda t}\mathbb{V}_2(t) \geq e^{-\lambda t}\underline{\eth}_{\mathbb{P}_{i\Im}}x^T(t)\mathbb{R}x(t) + \int_0^t x^T(\xi)(e^{-\lambda\xi}\mathbb{Z})x(\xi)d\xi + \int_0^t x^T(t_k)(e^{-\lambda t}\mathbb{Q})x(t_k)dt \tag{33}$$

By virtue of (32) and (33), we get

$$E\{x^T(t)\mathbb{R}x(t)\} \leq \frac{\bar{\eth}_{P_{i\Im}}d_1 + \spadesuit_0 + 4\tau^2\lambda\top^* + 4\lambda\phi^2\varsigma + \lambda\varsigma}{e^{-\lambda\top^*}\underline{\eth}_{\mathbb{P}_{i\Im}}} \tag{34}$$

Consequently, we can derive that $E\{x^T(t)\mathbb{R}x(t)\} < d^*$. According to **Definition 2.1**, the CLS with the SMC law is FTB about $(d_1, d^*, [0, \top^*], \mathbb{R}, \Omega_{[0,\top^*],\varsigma})$. Therefore, the theorem is proved completely.

3.4 Finite-time boundedness in $[\top^*, \top]$

In the time $[\top^*, \top]$, the period of sliding motion stage, which indicates that $s(t) = 0$ and $\dot{s}(t) = 0$ hold for this period. Then we can obtain equivalent sliding mode control law as follows:

$$u_{eq}(t) = \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j[-(\mathbb{J}B_{i\Im})^{-1}\mathbb{J}A_{i\Im}x(t) - (\mathbb{J}B_{i\Im})^{-1}\mathbb{J}D_{i\Im}w(t)] \tag{35}$$

By virtue of (29) and (2), we can get

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j[(I - \bar{B}_{i\Im})(A_{i\Im}x(t) + D_{i\Im}w(t))] \\ y(t) = C_\Im x(t) \end{cases} \tag{36}$$

wherein $\bar{B}_{i\Im} \triangleq B_{i\Im}(\mathbb{J}B_{i\Im})^{-1}\mathbb{J}$.

Theorem 3.3. *With respect to the SMC law (8), given parameters $d_2 > d_1 > 0, \lambda > 0, \varsigma > 0, \tau > 0, \kappa > 0, \rho > 0, \wp > 0, N, M$ and matrix $\mathbb{R} > 0$ are predetermined. If there are scalars $d^* > 0, \top^* > 0, \xi_1 > 0, \xi_2 > 0$, matrices $\mathbb{P}_{i\Im} > 0$ and symmetric matrices $L_\Im$, such that the following conidtions hold for all $i, j \in \digamma, \Im \in \P$*

$$\sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j \hat{W}_{ij} < 0 \tag{37}$$

$$\mathbb{P}_{i\pounds} - L_\Im \le 0, \pounds \in \P_{u\lambda}^{\Im}, \pounds \ne \Im \tag{38}$$

$$\mathbb{P}_{i\pounds} - L_\Im \ge 0, \pounds \in \P_{u\lambda}^{\Im}, \pounds = \Im \tag{39}$$

$$d_1 < d^* < d_2 \tag{40}$$

$$\frac{\overline{\eth}_{\mathbb{P}_{i\Im}} d^* + \spadesuit_0 e^{-(\kappa+\frac{1}{\rho})t} + \lambda\varsigma}{\underline{\eth}_{\mathbb{P}_{i\Im}}} < e^{-\lambda\top} d_2 \tag{41}$$

where

$$\hat{W}_{ij} = \begin{bmatrix} \hat{W}_{11} & \hat{W}_{12} & \hat{W}_{13} \\ * & \hat{W}_{22} & \hat{W}_{23} \\ * & * & \hat{W}_{33} \end{bmatrix} < 0$$

$$\hat{W}_{11} = \sum_{\pounds\in\P_{\lambda}^{\Im}} \varphi_{\Im\pounds}(\mathbb{P}_{i\pounds} - L_\Im) + \mathbb{P}_{i\Im}\tilde{A}_{i\Im} + \tilde{A}_{i\Im}^T\mathbb{P}_{i\Im} + (1+k_2)\wp N - \lambda\mathbb{P}_{i\Im}$$

$$\hat{W}_{12} = \begin{bmatrix} \mathbb{P}_{i\Im}D_{i\Im} & 0 & 0 \end{bmatrix}, \hat{W}_{13} = \begin{bmatrix} \xi_1\mathbb{P}_{i\Im}\bar{B}_{i\Im} & 0 & \xi_2\mathbb{P}_{i\Im}\bar{B}_{i\Im} & A_{i\Im}^T \end{bmatrix}$$

$$\hat{W}_{22} = -diag\begin{bmatrix} \lambda I & (1+k_2)M & \kappa - \frac{k_2}{\rho} + \lambda \end{bmatrix}, \hat{W}_{33} = -diag\begin{bmatrix} \xi_1 I & \xi_1 I & \xi_2 I & \xi_2 I \end{bmatrix}$$

$$\hat{W}_{23} = \Theta_1^T D_{i\Im}^T \Theta_2, \Theta_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}, \Theta_2 = \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}$$

then, the system (36) is FTB about $\left(d^*, d_2, [\top^*, \top], \mathbb{R}, \Omega_{[T^*,T],\varsigma}\right)$.

Proof: Pick the Lyapunov function $\mathbb{V}_3(x(t), \Im, t, \spadesuit(t)) = x^T(t)\mathbb{P}_{i\Im}x(t) + \spadesuit(t)$.

For simplicity, let $\mathbb{V}_3(t) \triangleq \mathbb{V}_3(x(t), \Im, t, \spadesuit(t))$, the weak infinitesimal generator of $\mathbb{V}_3(t)$ is given:

$$\Gamma\mathbb{V}_3(t) = x^T(t)\sum_{\pounds=1}^{R}\varphi_{\Im\pounds}\mathbb{P}_{i\pounds}x(t) + sym\{x^T(t)\mathbb{P}_{i\Im}\sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j[A_{i\Im}x(t) + D_{i\Im}w(t) - \bar{B}_{i\Im}A_{i\Im}x(t)$$
$$-\bar{B}_{i\Im}D_{i\Im}w(t)]\} - \kappa\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t)$$

In accordance with **Lemma 2.1**, we can readily obtain that

$$sym\{x^T(t)\mathbb{P}_{i\Im}\bar{B}_{i\Im}D_{i\Im}w(t)\} < \xi_1 x^T(t)\mathbb{P}_{i\Im}\bar{B}_{i\Im}\bar{B}_{i\Im}^T\mathbb{P}_{i\Im}x(t) + \xi_1^{-1}w^T(t)D_{i\Im}^T D_{i\Im}w(t),$$

$$sym\{x^T(t)\mathbb{P}_{i\Im}\bar{B}_{i\Im}A_{i\Im}x(t)\} < \xi_2 x^T(t)\mathbb{P}_{i\Im}\bar{B}_{i\Im}\bar{B}_{i\Im}^T\mathbb{P}_{i\Im}x(t) + \xi_2^{-1}x^T(t)A_{i\Im}^T A_{i\Im}x(t).$$

Similar to the certification process of **Theorem 3.2**, $\Gamma\mathbb{V}_3(t)$ can be rewritten as:

$$\begin{aligned} \Gamma\mathbb{V}_3(t) =& x^T\{\sum_{\pounds\in\P_{\lambda}^{\Im}}\varphi_{\Im\pounds}(\mathbb{P}_{i\pounds} - L_\Im) + \sum_{\pounds\in\P_{u\lambda}^{\Im}}\varphi_{\Im\pounds}(\mathbb{P}_{i\pounds} - L_\Im) + \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j[\mathbb{P}_{i\Im}\tilde{A}_{i\Im} + \tilde{A}_{i\Im}^T\mathbb{P}_{i\Im} \\ & - (\xi_1+\xi_2)\mathbb{P}_{i\Im}\bar{B}_{i\Im}\bar{B}_{i\Im}^T\mathbb{P}_{i\Im} - \xi_2^{-1}A_{i\Im}^T A_{i\Im}] + \wp N\}x(t) + sym\{x^T(t)\mathbb{P}_{i\Im}\sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j D_{i\Im}w(t)\} \\ & - \xi_1^{-1}w^T(t)D_{i\Im}^T D_{i\Im}w(t) - \kappa\spadesuit(t) - e^T(t)Me(t) \end{aligned} \tag{42}$$

Select the auxiliary function

$H_2(\Im, t, \spadesuit(t)) = \Gamma\mathbb{V}_3(t) - \lambda\mathbb{V}_3(t) - \lambda w^T(t)w(t)$

If the DET condition (3) is violated, for any scalar $k_2 > 0$, we can obtain from (42)

$$H_2(\Im, t, \spadesuit(t)) \le H_2(\Im, t, \spadesuit(t)) + k_2(\frac{1}{\rho}\spadesuit(t) + \wp x^T(t)Nx(t) - e^T(t)Me(t)) \tag{43}$$

Notice that $\varphi_{\Im \pounds} \geq 0$ for all $\Im \neq \pounds$ and $\varphi_{\Im\Im} = -\sum_{\pounds \neq \Im} \varphi_{\Im \pounds}$ for all $\Im \in \P_{\lambda}$, if $\Im \in \P_{\lambda}^{\Im}$ (the elements of the diagonal are known), by inequalities (37)-(38) and Schur complement, the following inequality holds:

$$\Gamma \mathbb{V}_3(t) < \lambda \mathbb{V}_3(t) + \lambda w^T(t) w(t) \tag{44}$$

If $\Im \in \P_{u\lambda}^{\Im}$ (the elements of the diagonal are unknown), according to inequalities (37)-(39) and Schur complement, the following (44) holds.
Employing $e^{-\lambda t}$ to multiply both sides of inequality (44), one gets that

$$\frac{e^{-\lambda t} d\mathbb{V}_3(t)}{dt} < \lambda e^{-\lambda t} \mathbb{V}_3(t) + \lambda e^{-\lambda t} w^T(t) w(t) \tag{45}$$

For $t \in [\top^*, \top]$, integrate the above inequality from $\top^*$ to t, which yields

$$\begin{aligned} e^{-\lambda t} \mathbb{V}_3(t) <& \mathbb{V}_3(\top^*) + \lambda \int_{\top^*}^{\top} e^{-\lambda \xi} w^T(\xi) w(\xi) d\xi \\ \leq& \overline{\eth}_{\mathbb{P}_{i\Im}} d^* + \spadesuit_0 e^{-(\kappa + \frac{1}{\rho})t} + \lambda \varsigma \end{aligned} \tag{46}$$

Besides, it is clear that

$$e^{-\lambda t} \mathbb{V}_3(t) \geq e^{-\lambda t} \underline{\eth}_{\mathbb{P}_{i\Im}} x^T(t) \mathbb{R} x(t) + e^{-\lambda t} \spadesuit(t) \tag{47}$$

By virtue of (46) and (47), we get

$$E\{x^T(t) \mathbb{R} x(t)\} \leq \frac{\overline{\eth}_{P_{i\Im}} d^* + \spadesuit_0 e^{-(\kappa + \frac{1}{\rho})t} + \lambda \varsigma}{e^{-\lambda \top} \underline{\eth}_{\mathbb{P}_{i\Im}}} \tag{48}$$

Consequently, in line with (41), we can derive that $E\{x^T(t) \mathbb{R} x(t)\} < d_2$ for $t \in [\top^*, \top]$.

According to **Definition 2.1**, the CLS with the SMC law is FTB about $\left(d^*, d_2, [\top^*, \top], \mathbb{R}, \Omega_{[\top^*, \top], \varsigma}\right)$.

Therefore, the theorem is proved completely.

Theorem 3.4. *For given parameters* $d_2 > d_1 > 0, \lambda > 0, \varsigma > 0, \wp > 0, \tau > 0, \kappa > 0, \rho > 0, k > 0, N, M, q_j$ *satisfying* $h_j - q_j \eta_j \geq 0$ $(q_j \in (0,1))$, *and matrix* $\mathbb{R} > 0$, *if there are scalars* $\varepsilon > 0, \xi_1 > 0, \xi_2 > 0, \top^* > 0, d^* > 0$, *matrices* $\mathbb{P}_{i\Im} > 0, \mathbb{Z} > 0, \mathbb{Q} > 0$, $\Phi_i = \Phi_i^T$ *with appropriate dimensions and symmetric matrices* $\tilde{L}_{\Im}$, *such that the following conditions hold for all* $i, j \in F, \Im \in \P$

$$\tilde{W}_{ij} - \Phi_i < 0, \Im \in \P_{\lambda}^{\Im} \tag{49}$$

$$\tilde{W}_{ij}^* - \Phi_i < 0, \Im \in \P_{u\lambda}^{\Im} \tag{50}$$

$$q_i \tilde{W}_{ii} - (1 - q_i)\Phi_i < 0, \Im \in \P_{\lambda}^{\Im} \tag{51}$$

$$q_i \tilde{W}_{ii}^* - (1 - q_i)\Phi_i < 0, \Im \in \P_{u\lambda}^{\Im} \tag{52}$$

$$q_j \tilde{W}_{ij} + (1 - q_j)\Phi_i + q_i \tilde{W}_{ji} + (1 - q_i)\Phi_j < 0, \Im \in \P_{\lambda}^{\Im} \tag{53}$$

$$q_j \tilde{W}_{ij}^* + (1 - q_j)\Phi_i + q_i \tilde{W}_{ji}^* + (1 - q_i)\Phi_j < 0, \Im \in \P_{u\lambda}^{\Im} \tag{54}$$

$$\begin{bmatrix} -\frac{e^{-\lambda \top} d^*}{2} + \Lambda_1 & \sqrt{d_1} \\ * & -\varepsilon \end{bmatrix} < 0 \tag{55}$$

$$\begin{bmatrix} -e^{-\lambda \top} \tilde{\mathbb{Q}} & \sqrt{4\lambda} \tilde{K}_{i\Im}^T \\ * & -I \end{bmatrix} < 0 \tag{56}$$

$$\begin{bmatrix} -e^{-\lambda \top} \mathbb{Z} & \sqrt{4\lambda \beta^2} \\ * & -I \end{bmatrix} < 0 \tag{57}$$

$$2d^* + 2\lambda\varsigma\varepsilon + 2\spadesuit_0 e^{-(\kappa+\frac{1}{\rho})t}\varepsilon \le e^{-\lambda T} d_2\varepsilon \tag{58}$$

$$d_1 < d^\star < d_2 \tag{59}$$

$$0 < \sum_{\pounds=1}^{R} \varphi_{\Im\pounds}(B_{i\Im}\tilde{K}_{i\pounds} + \tilde{K}_{i\pounds}^T B_{i\Im}^T) \tag{60}$$

$$\varepsilon\mathbb{R}^{-1} < \mathbb{P}_{i\Im} < 2\mathbb{R}^{-1} \tag{61}$$

$$\begin{bmatrix} -\tilde{L}_\Im & * \\ \tilde{\mathbb{P}}_{i\Im} & \tilde{\mathbb{P}}_{i\pounds} \end{bmatrix} \le 0, \pounds \in \P_{u\lambda}^{\Im}, \pounds \ne \Im \tag{62}$$

$$\tilde{\mathbb{P}}_{i\pounds} - \tilde{L}_\pounds \ge 0, \pounds \in \P_{u\lambda}^{\Im}, \pounds = \Im \tag{63}$$

where

$$\tilde{W}_{ij} = \begin{bmatrix} \tilde{W}_{11} & \tilde{W}_{12} & \tilde{W}_{13} & \tilde{W}_{14} \\ * & \tilde{W}_{22} & \tilde{W}_{23} & 0 \\ * & * & \tilde{W}_{33} & 0 \\ * & * & * & \tilde{W}_{44} \end{bmatrix} < 0, \tilde{W}_{ij}^* = \begin{bmatrix} \tilde{W}_{11}^* & \tilde{W}_{12} & \tilde{W}_{13} & \tilde{W}_{14} \\ * & \tilde{W}_{22} & \tilde{W}_{23} & 0 \\ * & * & \tilde{W}_{33} & 0 \\ * & * & * & \tilde{W}_{44} \end{bmatrix} < 0$$

$$\tilde{W}_{11} = \varphi_{\Im\Im}\tilde{\mathbb{P}}_{i\Im} + A_{i\Im}\tilde{\mathbb{P}}_{i\Im} + \tilde{\mathbb{P}}_{i\Im}A_{i\Im}^T + \tilde{\mathbb{Q}} + (1+k)\wp\tilde{N} + \sum_{\pounds=1}^{R}\varphi_{\Im\pounds}(B_{i\Im}\tilde{K}_{i\pounds} + \tilde{K}_{i\pounds}^T B_{i\Im}^T) - \sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}\tilde{L}_\Im - \lambda\tilde{\mathbb{P}}_{i\Im}$$

$$\tilde{W}_{11}^* = A_{i\Im}\tilde{\mathbb{P}}_{i\Im} + \tilde{\mathbb{P}}_{i\Im}A_{i\Im}^T + \tilde{\mathbb{Q}} + (1+k)\wp\tilde{N} + \sum_{\pounds=1}^{R}\varphi_{\Im\pounds}(B_{i\Im}\tilde{K}_{i\pounds} + \tilde{K}_{i\pounds}^T B_{i\Im}^T) - \sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}\tilde{L}_\Im - \lambda\tilde{\mathbb{P}}_{i\Im}$$

$$\tilde{W}_{12} = \begin{bmatrix} D_{i\Im} & -\sum_{\pounds=1}^{R}\varphi_{\Im\pounds}B_{i\Im}\tilde{K}_{i\pounds} - \tilde{\mathbb{Q}} & -B_{i\Im} & 0 \end{bmatrix}, \tilde{W}_{13} = \begin{bmatrix} \tilde{\mathbb{P}}_{i\Im} & \xi_1\bar{B}_{i\Im} & 0 & \xi_2\bar{B}_{i\Im} & \tilde{\mathbb{P}}_{i\Im}A_{i\Im}^T \end{bmatrix}$$

$$\tilde{W}_{14} = \begin{bmatrix} \tilde{\mathbb{P}}_{i\Im} & \cdots & \tilde{\mathbb{P}}_{i\Im} & \cdots\tilde{\mathbb{P}}_{i\Im} \end{bmatrix}, \tilde{W}_{22} = -diag\begin{bmatrix} \lambda I & (1+k)\tilde{M} - \tilde{\mathbb{Q}} & \lambda I & \kappa + \lambda - \frac{k}{\rho} \end{bmatrix}$$

$$\tilde{W}_{23} = \tilde{\Theta}_1^T D_{i\Im}^T \tilde{\Theta}_2, \tilde{\Theta}_1 = \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}, \tilde{\Theta}_2 = \begin{bmatrix} 0 & 0 & I & 0 & 0 \end{bmatrix}$$

$$\tilde{W}_{33} = -diag\begin{bmatrix} Z^{-1} & \xi_1 I & \xi_1 I & \xi_2 I & \xi_2 I \end{bmatrix}$$

$$\tilde{W}_{44} = -diag\begin{bmatrix} \varphi_{\Im\lambda_1^\Im}^{-1}\tilde{\mathbb{P}}_{i\lambda_1^\Im} & \cdots & \varphi_{\Im\lambda_l^\Im}^{-1}\tilde{\mathbb{P}}_{i\lambda_l^\Im} & \cdots & \varphi_{\Im\lambda_g^\Im}^{-1}\tilde{\mathbb{P}}_{i\lambda_g^\Im} \end{bmatrix}_{\Im\ne\lambda_l^\Im}$$

$\Lambda_1 = 4\tau^2\lambda\top + 4\lambda\phi^2\varsigma + \lambda\varsigma + \spadesuit_0, \tilde{\mathbb{Q}} = \tilde{\mathbb{P}}_{i\Im}\mathbb{Q}\tilde{\mathbb{P}}_{i\Im}, \tilde{\mathbb{M}} = \tilde{\mathbb{P}}_{i\Im}\mathbb{M}\tilde{\mathbb{P}}_{i\Im}, \tilde{\mathbb{N}} = \tilde{\mathbb{P}}_{i\Im}\mathbb{N}\tilde{\mathbb{P}}_{i\Im}, \tilde{\mathbb{L}}_\Im = \tilde{\mathbb{P}}_{i\Im}\mathbb{L}_\Im\tilde{\mathbb{P}}_{i\Im}$.

then, the resultant CLS is FTB about $(d_1, d_2, [0,\top], \mathbb{R}, \Omega_{[0,T],\varsigma})$. And the gains of sliding mode controller are obtained as $K_{i\Im} = \tilde{K}_{i\Im}\tilde{P}_{i\Im}^{-1}$.

Proof. As for the transition probabilities φ in $\tilde{W}_{11}$, it can be divided into two parts:

$$\tilde{\mathbb{P}}_{i\Im}(\sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}(\tilde{\mathbb{P}}_{i\pounds} - L_\Im))\tilde{\mathbb{P}}_{i\Im} = \begin{cases} \varphi_{\Im\Im}\tilde{\mathbb{P}}_{i\Im} + \tilde{\mathbb{P}}_{i\Im}(\sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}\tilde{\mathbb{P}}_{i\pounds})\tilde{\mathbb{P}}_{i\Im} - \tilde{\mathbb{P}}_{i\Im}(\sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}L_\Im)\tilde{\mathbb{P}}_{i\Im}, & \Im \in \P_\lambda^\Im, \\ \tilde{\mathbb{P}}_{i\Im}(\sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}\tilde{\mathbb{P}}_{i\pounds})\tilde{\mathbb{P}}_{i\Im} - \tilde{\mathbb{P}}_{i\Im}(\sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}L_\Im)\tilde{\mathbb{P}}_{i\Im}, & \Im \in \P_{u\lambda}^\Im. \end{cases} \tag{64}$$

We can apparently deduce that the following inequality (65) can ensure that the establishment of conditions (17) and (37) in the finite-time phase $[0,\top]$

$$\sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j \breve{W}_{ij} < 0 \tag{65}$$

where

$$\breve{W}_{ij} = \begin{bmatrix} \breve{W}_{11} & W_{12} & \breve{W}_{13} \\ * & \breve{W}_{22} & \tilde{W}_{23} \\ * & * & \breve{W}_{33} \end{bmatrix}$$

$$\breve{W}_{11} = \sum_{\pounds\in\P_\lambda^\Im}\varphi_{\Im\pounds}(\mathbb{P}_{i\pounds} - L_\Im) + \mathbb{P}_{i\Im}\tilde{A}_{i\Im} + \tilde{A}_{i\Im}^T\mathbb{P}_{i\Im} + \mathbb{Q} + (1+k)\wp N - \lambda\mathbb{P}_{i\Im}$$

$\breve{W}_{13} = \begin{bmatrix} I & \xi_1 \mathbb{P}_{i\Im} \bar{B}_{i\Im} & 0 & \xi_2 \mathbb{P}_{i\Im} \bar{B}_{i\Im} & A_{i\Im}^T \end{bmatrix}, \breve{W}_{22} = -diag\left[\lambda I, (1+k)M - \mathbb{Q}, \lambda I, \kappa - \frac{k}{\rho} + \lambda\right]$

$\breve{W}_{33} = -diag\begin{bmatrix} Z^{-1} & \xi_1 I & \xi_1 I & \xi_2 I & \xi_2 I \end{bmatrix}$

In the following, to simplify the notation, let $\tilde{\mathbb{P}}_{i\Im} \triangleq \mathbb{P}_{i\Im}^{-1}$, and perform congruent transformation to inequality (65) with $diag\left\{\tilde{\mathbb{P}}_{i\Im}, I, \tilde{\mathbb{P}}_{i\Im}, I, I, I, I, I, I, I\right\}$. Then, in line with (64) and Schur complement, we derive that $\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \tilde{W}_{ij} < 0$ and $\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \tilde{W}_{ij}^* < 0$.

So as to make full use of MFs information and reduce the conservativeness, slack matrices Φ_i are introduced. By using the property of MFs, i.e., $\sum_{i=1}^{v} \eta_i(x(t)) = 1$,we obtain:

$$\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i(\eta_j - h_j)\Phi_i = \sum_{i=1}^{v} \eta_i(\sum_{j=1}^{v} \eta_j - \sum_{j=1}^{v} h_j)\Phi_i = 0 \tag{66}$$

According to $\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \tilde{W}_{ij} < 0$ and (54), we can get that:

$$\begin{aligned}
&\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \tilde{W}_{ij} \\
&= \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \tilde{W}_{ij} + \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i(\eta_j - h_j)\Phi_i \\
&= \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j(\tilde{W}_{ij} - \Phi_i) + \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i \eta_j \Phi_i \\
&= \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i(h_j - q_j\Phi_j)(\tilde{W}_{ij} - \Phi_i) + \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i \eta_j (q_j(\tilde{W}_{ij} - \Phi_i) + \Phi_i) \\
&= \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i(h_j - q_j\eta_j)(\tilde{W}_{ij} - \Phi_i) + \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i \eta_j (q_j\tilde{W}_{ij} + (1 - q_j)\Phi_i) \\
&= \sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i(h_j - q_j\eta_j)(\tilde{W}_{ij} - \Phi_i) + \sum_{i=1}^{v} \eta_i^2 (q_i\tilde{W}_{ii} + (1 - q_i)\Phi_i) \\
&\quad + \sum_{i=1}^{v}\sum_{j>i}^{v} \eta_i \eta_j [q_j\tilde{W}_{ij} + (1 - q_j)\Phi_i + q_i\tilde{W}_{ji} + (1 - q_i)\Phi_j]
\end{aligned} \tag{67}$$

Combined with $\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \tilde{W}_{ij} < 0$, we can get that inequalities (49),(51),(53) hold. Similar to the above certification process, we can obtain that (50),(52),(54) can be ensured by $\sum_{i=1}^{v}\sum_{j=1}^{v} \eta_i h_j \tilde{W}_{ij}^* < 0$.

Besides, in accordance with (61), one gets:

$$\alpha_{max}\left(\mathbb{R}^{-\frac{1}{2}} \mathbb{P}_{i\Im} \mathbb{R}^{\frac{1}{2}}\right) < \frac{1}{\varepsilon}, \alpha_{min}\left(\mathbb{R}^{-\frac{1}{2}} \mathbb{P}_{i\Im} \mathbb{R}^{\frac{1}{2}}\right) > \frac{1}{2} \tag{68}$$

Furthermore, it is obvious that (20),(21),(23),(41) can be guaranteed by (57),(56),(55),(58) respectively.

Finally, (62)-(63) can be deduced by pre-multiplying and post-multiplying simultaneously by $\mathbb{P}_{i\Im}^{-1}$ with (18)-(19) or (38)-(39), then the proof is over.

Theorem 3.5. *With regard to the controlled interval type-II fuzzy MJSs (1), the DET condition (3) can assure that the triggering interval* $\blacksquare = t_{k+1} - t_k$, $k \in N$ *satisies*

$$\blacksquare \geq \frac{1}{a_1} ln\left[1 + \frac{a_1(\spadesuit(t_{k+1}) + \rho\wp x^T(t)Nx(t))}{a_2\rho\alpha_{max}(M)}\right] \triangleq \veebar \tag{69}$$

where
$a_1 = \left\|\tilde{A}\right\| + \left\|\tilde{B}K_{i\Im}\right\| + \gamma(t)\left\|\tilde{B}\right\| + \left\|\tilde{D}\right\|,$
$a_2 = \left\|\tilde{A}\right\|\|x(t)\|^2 + \left\|\tilde{B}K_{i\Im}\right\|\|x(t_k)\|^2 + \gamma(t)\left\|\tilde{B}\right\| + \left\|\tilde{D}\right\|\bar{w}^2,$
with
$\tilde{A} \triangleq \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j A_{i\Im}, \tilde{B} \triangleq \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j B_{i\Im}, \tilde{D} \triangleq \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j D_{i\Im}.$
Thus, there is a positive scalar $\veebar$ to avoid the Zeno phenomenon.
Proof. With respect to $\forall t \in [t_k, t_{k+1})$, we get

$$\begin{aligned}\dot{e}(t) =& \sum_{i=1}^{v}\eta_i(x(t))[A_{i\Im}x(t) + B_{i\Im}u(t) + D_{i\Im}w(t)]\\ =& \sum_{i=1}^{v}\sum_{j=1}^{v}\eta_i h_j[A_{i\Im}x(t) + B_{i\Im}K_{i\Im}x(t_k) - B_{i\Im}\gamma(t)sgn(s(t)) + D_{i\Im}w(t)]\\ =& \tilde{A}x(t) + \tilde{B}K_{i\Im}x(t_k) - \tilde{B}\gamma(t)sgn(s(t)) + \tilde{D}w(t)\end{aligned} \tag{70}$$

Moreover, we can deduce that from (70)

$$\begin{aligned}\tfrac{d}{dt}\|e(t)\|^2 &\leq 2\|e(t)\|\|\dot{e}(t)\|\\ &\leq \left\|\tilde{A}\right\|(\|x(t)\|^2 + \|e(t)\|^2) + \left\|\tilde{B}K_{i\Im}\right\|(\|x(t_k)\|^2 + \|e(t)\|^2) + \gamma(t)\left\|\tilde{B}\right\|(1 + \|e(t)\|^2)\\ &\quad + \left\|\tilde{D}\right\|(\bar{w}^2 + \|e(t)\|^2)\\ &= \left[\left\|\tilde{A}\right\| + \left\|\tilde{B}K_{i\Im}\right\| + \gamma(t)\left\|\tilde{B}\right\| + \left\|\tilde{D}\right\|\right]\|e(t)\|^2 + \left\|\tilde{A}\right\|\|x(t)\|^2\\ &\quad + \left\|\tilde{B}K_{i\Im}\right\|\|x(t_k)\|^2 + \gamma(t)\left\|\tilde{B}\right\| + \bar{w}^2\left\|\tilde{D}\right\|\\ &\triangleq a_1\|e(t)\|^2 + a_2\end{aligned}$$

In line with Comparison Lemma and $e(t_k) = 0$ at the triggering instant, one gets from $\frac{d}{dt}\|e(t)\|^2 \leq a_1\|e(t)\|^2 + a_2$

$$\|e(t_{k+1})\|^2 \leq \frac{a_2}{a_1}(e^{a_1(t_{k+1}-t_k)} - 1) \tag{71}$$

In accordance with the dynamic event-triggered condition (3), we get

$$\frac{\spadesuit(t_{k+1}) + \rho\wp x^T(t_{k+1})Nx(t_{k+1})}{\rho\alpha_{max}(M)} \leq \|e(t_{k+1})\|^2 \tag{72}$$

By virtue of (71) and (72), we can get the triggering interval

$$\blacksquare \geq \frac{1}{a_1}ln[1 + \frac{a_1(\spadesuit(t_{k+1}) + \rho\wp x^T(t_{k+1})Nx(t_{k+1}))}{a_2\rho\alpha_{max}(M)}] \triangleq \veebar \tag{73}$$

Due to $\rho > 0$, which means that the term $a_1(\spadesuit(t_{k+1}) + \rho\wp x^T(t_{k+1})Nx(t_{k+1}))$ is strictly greater than zero. Therefore, by means of (73), we can derive that $\veebar > 0$. Thus, the Zeno phenomenon can be excluded.

4 Numerical Examples

In the chapter, our goal is to verify the validity of the presented control plan via a numerical instance.

Consider a two-rule interval type-II fuzzy MJSs with three modes, which means $R = 3$. the partially known transition rate matrix is given by

$$\aleph = \begin{bmatrix}-1 & ? & ?\\ 0.2 & -1 & 0.8\\ 0.5 & ? & ?\end{bmatrix}$$

The correlative system parameters are listed as follows:

$$A_{11} = \begin{bmatrix} -0.4875 & 2.0001 & 0.1137 \\ 0.2024 & -0.3162 & -3.0787 \\ -0.1105 & 0.3002 & -3.4147 \end{bmatrix}, A_{12} = \begin{bmatrix} -0.4731 & 2.1113 & 0.0747 \\ 0.1813 & -0.2517 & -3.0505 \\ -0.1201 & 0.4112 & -3.2123 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} -0.3831 & 1.8525 & 0.0509 \\ 0.1926 & -0.2817 & -4.0525 \\ -0.1201 & 0.4331 & -4.2138 \end{bmatrix}, A_{21} = \begin{bmatrix} -0.5001 & 1.3714 & 0.1546 \\ 0.0723 & -0.2312 & -2.0426 \\ -0.1434 & 0.5161 & -2.3935 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -0.7215 & 1.4515 & 0.1815 \\ 0.3321 & -0.5117 & -2.8520 \\ -0.2226 & 0.6487 & -2.5681 \end{bmatrix}, A_{23} = \begin{bmatrix} -0.7018 & 1.3515 & 0.1515 \\ 0.2219 & -0.4223 & -2.5530 \\ -0.4012 & 0.3330 & -2.6667 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} 1.2 \\ 1.6 \\ 0.8 \end{bmatrix}, B_{12} = \begin{bmatrix} 1.3 \\ 1.7 \\ 0.9 \end{bmatrix}, B_{13} = \begin{bmatrix} 1.4 \\ 1.1 \\ 0.4 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.3 \\ 0.8 \\ 0.7 \end{bmatrix}, B_{22} = \begin{bmatrix} 0.4 \\ 0.5 \\ 0.1 \end{bmatrix}, B_{23} = \begin{bmatrix} 0.8 \\ 1.2 \\ 0.9 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.41 & 0.17 & 0.31 \end{bmatrix}, C_2 = \begin{bmatrix} -0.54 & 0.82 & 0.33 \end{bmatrix}, C_3 = \begin{bmatrix} -0.75 & 0.38 & 0.62 \end{bmatrix},$$

$$\mathbb{J}_1 = \begin{bmatrix} 0.1550 & 0.2750 & 0.4875 \end{bmatrix}, \mathbb{J}_2 = \begin{bmatrix} 0.3700 & 0.4120 & 0.6300 \end{bmatrix}, \mathbb{J}_3 = \begin{bmatrix} 0.5327 & 0.4103 & 0.4993 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0.32 \\ 0.41 \\ 0.25 \end{bmatrix}, D_{12} = \begin{bmatrix} 0.11 \\ 0.09 \\ 0.17 \end{bmatrix}, D_{13} = \begin{bmatrix} 0.62 \\ 0.87 \\ 0.55 \end{bmatrix}, D_{21} = \begin{bmatrix} 0.35 \\ 0.16 \\ 0.21 \end{bmatrix}, D_{22} = \begin{bmatrix} 0.16 \\ 0.23 \\ 0.08 \end{bmatrix}, D_{23} = \begin{bmatrix} 0.39 \\ 0.32 \\ 0.61 \end{bmatrix}$$

with $w(t) = 0.32sin(0.6t)$,the upper with lower membership functions:

$$\underline{\eta}_1(x) = 0.75(1 - \frac{1}{1+e^{-(x_1+\frac{13}{2})/8}}), \overline{\eta}_1(x) = 0.75(1 - \frac{1}{1+e^{-(x_1+\frac{9}{2})/8}})$$

$$\underline{\eta}_2(x) = 1 - \underline{\eta}_1(x), \overline{\eta}_2(x) = 1 - \overline{\eta}_1(x)$$

and the weighting coefficients $\underline{\ell}_1 = sin^2(x_1), \overline{\ell}_1 = 1 - sin^2(x_1)$. The actual membership functions are expressed as $\eta_1(x) = \underline{\eta}_1(x)\underline{\ell}_1 + \overline{\eta}_1(x)\overline{\ell}_1$ and $\eta_2(x) = 1 - \eta_1(x)$. To perform the emulation, the starting location is selected as $x(0) = [-2.03, -2.52, -2.70]^T$.

Table 1. Distinct control schemes

Control scheme	$K_{i\Im}$	Triggering number
	$K_{11} = [-5.5221, -2.5181, -2.1123]$	
	$K_{12} = [-4.3245, -1.4781, -2.0388]$	
DETS	$K_{13} = [-2.0326, -3.6149, -1.7143]$	30
	$K_{21} = [-2.3704, -3.2247, -1.5427]$	
	$K_{22} = [-4.0126, -1.4281, -1.2827]$	
	$K_{23} = [-3.5186, -4.3135, -1.9017]$	
	$K_{11} = [-11.2327, -17.3302, -14.5187]$	
	$K_{12} = [-7.3431, -10.7341, -15.3162]$	
	$K_{13} = [-7.0326, -18.5249, -13.6234]$	
SETS	$K_{21} = [-11.1959, -22.2141, -25.2029]$	117
	$K_{22} = [-9.2504, -13.7324, -18.3048]$	
	$K_{23} = [-12.6203, -8.2219, -21.1703]$	

Therefore, the parameters can be chosen as $\xi_1 = 0.25, \xi_2 = 0.35, \varsigma = 0.6, \wp = 0.5, \kappa = 0.2, \rho = 0.6, \tau = 0.4, \varepsilon = 0.3, N = 1, M = 10, q_i = 0.16; 0.27(i = 1, 2), q_j = 0.05; 0.36(j = 1, 2), d_1 = 0.1, d_2 = 4.6, T = 5s$.

The simulation results are shown in Fig.2-Fig.6 and Table 1. From Fig.2, we can see that the trajectories of the sliding variable can be compelled into the sliding surface $s(t) = 0$ at $t = \top^*, \top^* < \top$. And stays on the sliding surface for the rest of the time $[\top^*, \top]$. Fig.3 depicts the SMC law, where the resultant CLS with the proposed event-triggered SMC law u(t) is stable. From Fig.4, we can observe that the states of the CLS can be forced by the designed controller, and then it arrives onto the sliding surface as depicted in Fig.2 and finally converges to the steady-state. Fig.5 and Fig.6 displays transmit instants and transmit intervals of the system under DETS and SETS, respectivlely. Besides, Table 1 shows that the triggering number under the DETS is less than the one in the SETS . In other words, the triggering under the SETS is more frequent than the one under the DETS. Summarizing the aforementioned simulation results, the effectiveness of the proposed method has been confirmed.

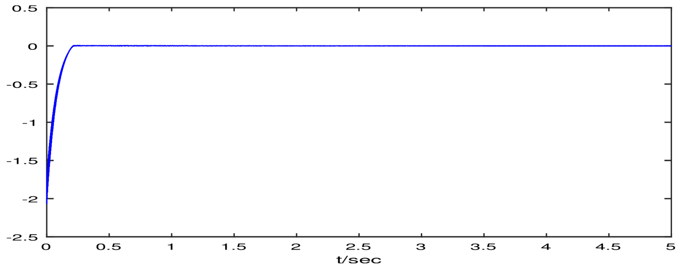

Fig. 2. Sliding surface $s(t)$ (Case of partially known transition probabilities).

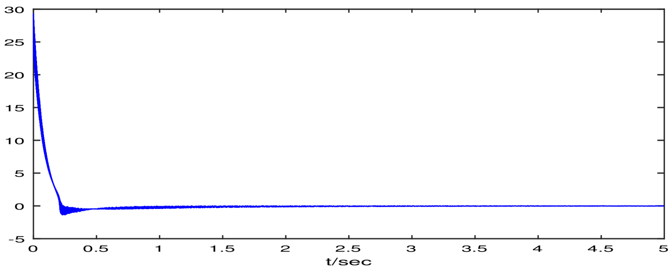

Fig. 3. Control input $u(t)$ (Case of partially known transition probabilities).

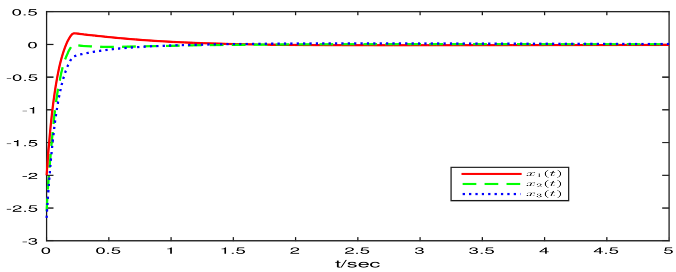

Fig. 4. State response of the closed-loop system (Case of partially known transition probabilities).

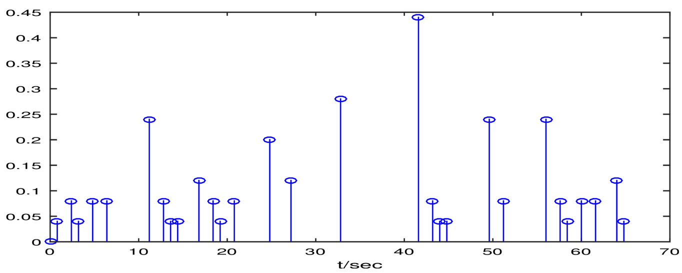

Fig. 5. Transmit instants and transmit intervals under dynamic event-triggerd scheme

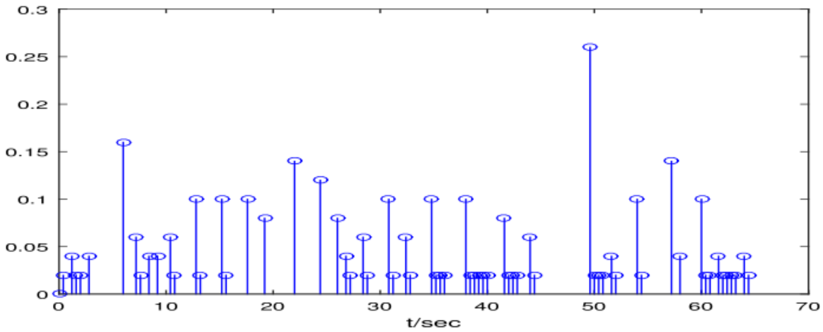

Fig. 6. Transmit instants and transmit intervals under static event-triggered scheme

5 Conclusions

In the article, the issue of finite-time event-triggered sliding mode control (SMC) is investigated for a class of interval type-II fuzzy Markov jump systems with partially known transition probabilities. For the sake of saving network resources, a dynamic event-triggered scheme (DETS) is proposed to determine whether to transmit the signal or not. The mismatched membership functions between the system and the controller are solved, and we obtain less conservative stability conditions. Then, a suitable fuzzy SMC law is devised that makes the state trajectory of the system reach the specified sliding surface in finite-time. Thereafter, the sufficient conditions of finite-time boundedness(FTB) in the reaching phase and sliding phase are derived by the time partition strategy. Besides, the suitable fuzzy controller has been computed. Finally, an example is presented to validate the effectiveness of the proposed control scheme. In the future, we will extend the theory to multiagent systems and combine it with distinct types of event-triggered schemes, such as resilient adaptive event-triggered scheme.

Acknowledgement

This work was supported in part by the National Natural Science Foundation of China under Grants 11661028; the Natural Science Foundation of Guangxi, PR China under Grants 2020GXNSFAA159141.

Competing Interests

Authors have declared that no competing interests exist.

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