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Estimation of the Residuals Entropy Function of Inverse Weibull Distribution Based on Generalized Type-II Hybrid Censored Samples

Moshera A. M. Ahmad[1*]

[1]El Gazeera High Institute for Computer and Management Information System, Egypt.

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

Shannon's entropy plays important role in the information theory. However, it can't be applied to systems which have survived for some time. Therefore, the concept of residual entropy was developed. In this paper, the estimation of the entropy of a two-parameter inverse Weibull distribution based on the generalized type-II hybrid censored sample is considered. The Bayes estimator for the residual entropy of the Inverse Weibull distribution under the generalized type-II hybrid censored sample is given. Simulation experiments are conducted to see the effectiveness of the different estimators.

Keywords: *Bayes estimation; entropy; inverse weibull distribution; generalized hybrid censoring; maximum likelihood estimation; residual entropy.*

1 Introduction

There is a message (or more) in any communication channel, the sender hope to send it to the receiver. If the channel is perfect the message will arrive complete. But most likely, the channel suffers from a lot of noise such as bad line, data jam, etc. Then, we may need to measure how perfect communication over (through) an

**Corresponding author: E-mail: moshera_ahmad@pg.cu.edu.eg, moshera1999@yahoo.com;*

imperfect communication channel. In other words, we need to be sure that the information which the message has carried is received completely. Entropy is a useful measure of uncertainty and dispersion, and it has many uses in communication theory. An early definition of information entropy was introduced by Shannon in [1], and it is usually referred to as Shannon's entropy.

Let X be a random variable with cumulative distribution function (cdf) $F(x)$, and probability density function (pdf) $f(x)$, then the entropy H_X of the random variable X is defined as:

$$H_X = H(f) = -E[\ln f(x)] = \int_{-\infty}^{\infty} f(x)\log(f(x))dx. \tag{1}$$

In this context, H_X is a measure of the uncertainty associated with the probability density function f. The Shannon's entropy plays a vital role as a measure of uncertainty in different areas such as physics, electronics, engineering, and economics.

Many authors worked on entropy's estimation for different distributions. Cramer and Bagh in [2] discussed the entropy of Weibull distribution under progressive censoring. Cho et al. in [3] presented an estimator for the entropy function of Rayleigh distribution based on doubly-generalized type II hybrid censored samples. Cho et al. in [4] considered the estimation of the entropy of Weibull distribution based on the generalized progressively censored sample. Ahmad in [5] derived the estimators for the entropy function of the Fréchet distribution under generalized type I hybrid censored samples. Mahmoud et al. in [6] derived the estimators for the entropy function of the Lomax distribution under generalized type I hybrid censored samples.
Consider an inverse Weibull distribution with cdf:

$$F(x;\alpha,\lambda) = e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}, x > 0, \alpha > 0, \lambda > 0, \tag{2}$$

and pdf:

$$f(x;\alpha,\lambda) = \alpha\lambda^{\alpha}x^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}, x > 0, \alpha > 0, \lambda > 0. \tag{3}$$

For the pdf (3), the entropy (1) simplifies to

$$H(f) = \gamma\left(1+\frac{1}{\lambda}\right) + \log\left(\frac{\alpha}{\lambda}\right) + 1 \tag{4}$$

where γ is the Euler-Mascheroni constant.

In the context of information theory, Shannon's information measure is useful for measuring the uncertainty associated with some density function. However, this entropy is not useful for a system that has survived for some units of time. It means that, there are some units that have low uncertainty and others that have great uncertainty. Then, if the random variable X represents the lifetime of a device, the characteristic of special interest is the residual life distribution, which is the distribution of the random variable $(X - t)$ truncated at $(t \geq 0)$. In other words, if a unit of life length X is known to have survived to age t, it is the residual entropy of $(X - t)$ that is of interest. Ebrahimi in [7] defines the residual entropy of a random variable X with density function f as

$$H(f,t) = -\int_{x=t}^{\infty} \frac{f(x)}{S(t)}\, log \frac{f(x)}{S(t)} dx; \qquad S(t) \geq 0 \tag{5}$$

Where $S(t)$ is the survival function of X. Using the relationship between the survival function and hazard function $h(x)$, the residual entropy function can be expressed as

$$H(f,t) = 1 - \frac{1}{S(t)}\int_{t}^{\infty} f(x)\log(h(x))dx. \tag{6}$$

In a lifetime experiment, it is most likely that the researcher terminates the experiment before the failure of all items. This is because of the waiting time for the last failure is unknown or that the items under study may be expensive. For these reasons the experimenter terminates the experiment before the last failure, and the data samples obtained from such situation are called censored samples. There are many types of censoring schemes. If we terminate the experiment at a fixed a pre-determined time T, we say that we have "type I censoring scheme". If we terminate the experiment at the r^{th} failure, we say that we have "type II censoring scheme". In the reliability literature, two mixtures of both these censoring schemes have been discussed under the title "hybrid censoring schemes" (HCS). If the experiment terminates when either the pre-fixed number of failures (r) has failed or a pre-specified censoring time T has been reached, this is called type I hybrid censoring scheme (Type-I HCS). We express the termination time of the experiment as $T_* = min\{X_{r:n}, T\}$. If the experiment terminates when either the last of a pre-fixed failure numbers has failed or a pre-specified censoring time T is reached, this is called type II hybrid censoring scheme (Type-II HCS). We express the termination time of the experiment as $T^* = max\{X_{r:n}, T\}$. However, in type I hybrid censoring, there is high probability that the pre-fixed time T occurs before obtaining enough failures times to make inference. on other side, in type II hybrid censoring, we might take a long time to observe the desired number of failures. To overcome these disadvantages, Chandrasekar et al. in [8] introduced generalized type I and type II hybrid censoring schemes.

Many authors have studied residual entropy function in different aspects. Ebrahimi and Pellerey in [9] proposed the Shannon residual entropy function as a measure of uncertainty. Belzunce et al. in [10] considered the residual entropy function. Drissi et al. in [11] consider the cumulative residual entropy. Baig and Dar in [12] studied the concept of Varma's entropy for the life time distributions that generalizes the entropy measure. Kayal in [13] studied a generalized residual entropy of record values and weighted distributions. Rajesh et al. in [14] proposed the local linear estimators for the conditional residual entropy function in the case of complete and censored samples.

In this paper, under the generalized type II hybrid censoring scheme (G-Type-II HCS), we derive and estimate the entropy and residual entropy of the inverse Weibull distribution. Also, we study the performance of the estimates using simulated data. The simulation contains different parameter values. The relative absolute bias and relative root MSE of the estimates have been obtained to assess the performance of the various estimates under different models. The rest of the paper is organized as follows; in section 2, we derive the residual entropy function associated with the Inverse Weibull model. In section 3, we discuss estimating the parameter of the inverse Weibull distribution under the G-Type-II HCS. In section 4, the maximum likelihood estimates of the entropy of the inverse Weibull distribution under G-Type-II HCS are obtained. In section 5, we derive the Bayes estimators for the residual entropy of an inverse Weibull distribution under the squared error loss (SEL) function. In section 6, some simulation studies are performed. Finally, the conclusions in section 7.

2 Estimation of the Residual Entropy Function of Inverse Weibull Distribution

The residual entropy measures the uncertainty contained in the conditional density of $(X - t)$ given $X > t$ about the predictability of remaining lifetime of the component. Moreover, $-\infty < H(f, t) < \infty$, and if $t = 0$ the residual entropy reduces to Shnnons's entropy which is defined over $(0, \infty)$, [see Pathiyil in [15].

Consider an inverse Weibull distribution with the pdf (3), survival function

$$S(x; \alpha, \lambda) = 1 - e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}, \tag{7}$$

and hazard function

$$h(x; \alpha, \lambda) = \frac{\alpha\lambda^{\alpha}x^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}}{1 - e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}}. \tag{8}$$

Then the residual entropy function associated with the inverse Weibull model is

$$H_{IW} = 1 - \frac{1}{\left(1 - e^{-\left(\frac{\lambda}{t}\right)^{\alpha}}\right)} \int_t^{\infty} \left(\alpha\lambda^{\alpha} x^{-(\alpha+1)} e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}\right) log \left[\frac{\alpha\lambda^{\alpha} x^{-(\alpha+1)} e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}}{1 - e^{-\left(\frac{\lambda}{x}\right)^{\alpha}}}\right] dx.$$

After some calculations the residual entropy function associated with the inverse Weibull model is

$$\begin{aligned} H_{IW} = 1 - \frac{1}{(1-\tau)} \Bigg[& -\tau(log\alpha + \alpha log\lambda) \\ & - (\alpha+1)\left\{-(1-\tau)log\lambda - \frac{1}{\alpha}\left(-\mathrm{Y} - \gamma\left(0, \left(\frac{\lambda}{t}\right)^{\alpha}\right) - \tau log\left(\frac{\lambda}{t}\right)^{\alpha}\right)\right\} + \gamma\left(2, \left(\frac{\lambda}{t}\right)^{\alpha}\right) \\ & + \{(1-\tau)\log(1-\tau) + \tau\}\Bigg], \end{aligned} \tag{9}$$

where $\tau = e^{-\left(\frac{\lambda}{t}\right)^{\alpha}}$, Y is Euler's constant (≈ 0.577), and $\gamma(s,z) = \int_0^z t^{s-1} e^{-t}\, dt$ is the lower incomplete gamma function.

3 Generalized Type-II Hybrid Censoring

Consider a life-testing experiment with n identical units placed on a life-test at time 0. Assume that $X_1, X_2, \dots, X_n$ denote the corresponding lifetimes from a distribution with cdf $F(x)$ and pdf $f(x)$. A G-Type-II HCS is described as follows; Fix an integer $r \in \{1, 2, \dots, n\}$ and fixed time points T_1 and $T_2 \in (0, \infty)$ such that $T_1 < T_2$. If the r^{th} failure occurs before time point T_1, terminate the experiment at T_1. If the r^{th} failure occurs between T_1 and T_2 terminate the experiment at the time of the failure, $X_{r:n}$. If the r^{th} failure occurs after time T_2, terminate the experiment at T_2. This type of censoring, while shooting for a minimum number of failures, r, guarantees that the experiment will be completed by time T_2. Thus T_2 serves as the absolute maximum time that the experiment would not be allowed to go beyond time T_2 [see, Balakrishnan and Kundu in [16]. In other words;

- If the r^{th} failure occurs before time T_1 , terminate the experiment at T_1,
- If the r^{th} failure occurs between time T_1, and time T_2 terminate the experiment at X_r,
- If the r^{th} failure occurs after time T_2, terminate the experiment at T_2.

In this type of HCS, the maximum time for the duration of the experiment is pre-fixed by T_2, and this is an advantage from an experiment's points view. We will observe one of the following forms of observations, under such a G-Type-II HCS:

$$Case\ I: \{x_{1:n} < x_{2:n} < \cdots < x_{r:n} < \cdots < x_{d_1} \le T_1)\}, \quad if\ x_{r:n} < T_1,$$

$$Case\ II: \{x_{1:n} < x_{2:n} < \cdots < T_1 < \cdots < x_{r:n})\}, \qquad if\ T_1 < x_{r:n} < T_2,$$

$$Case\ III: \{x_{1:n} < x_{2:n} < \cdots < T_1 < \cdots < x_{d_2} \le T_2\}, \qquad if\ x_{r:n} > T_2.$$

A schematic representation of the G-Type-II HCS is presented in Fig. 1.

Let d_1 and d_2 be the number of observed failures up to time points T_1 and T_2 respectively. Then, under a generalized type-II hybrid censored sample, the likelihood functions for the three different cases describe above are as follows:

Case I

$$\frac{n!}{(n-d_1)!} \prod_{i=1}^{d_1} f(x_{i:n})\, [S(T_1)]^{n-d_1}; \ \text{for}\ d_1 = r, (r+1), \dots, \text{or}\ n,$$

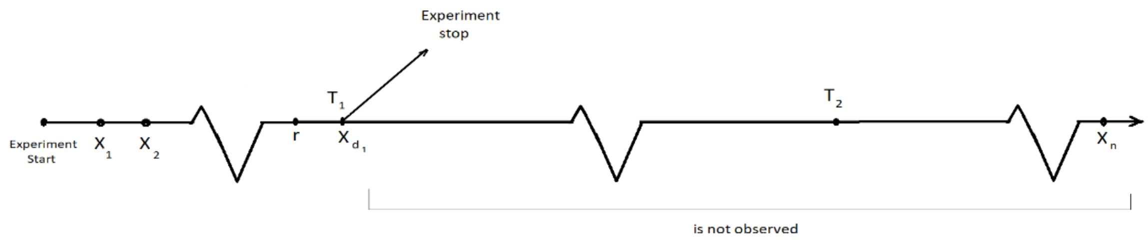
Case II

$$\frac{n!}{(n-r)!}\prod_{i=1}^{r} f(x_{\mathrm{i}:r}\,)\,[S(x_r)]^{n-r},$$

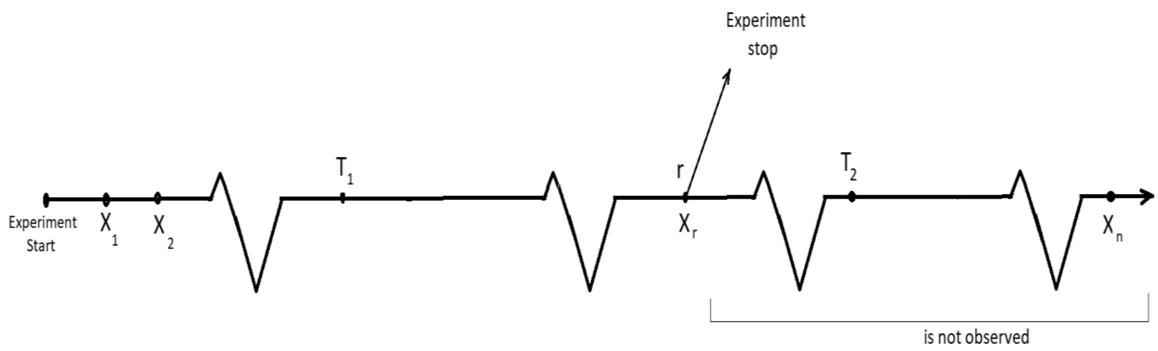
Case III

$$\frac{n!}{(n-d_2)!}\prod_{i=1}^{d_2} f(x_{\mathrm{i}:n}\,)\,[S(T_2)]^{n-d_2};\ \text{for}\ \ d_2=0,1,2,\dots,\text{or}\ (\,r-1)\,.$$

Case I



Case II



Case III

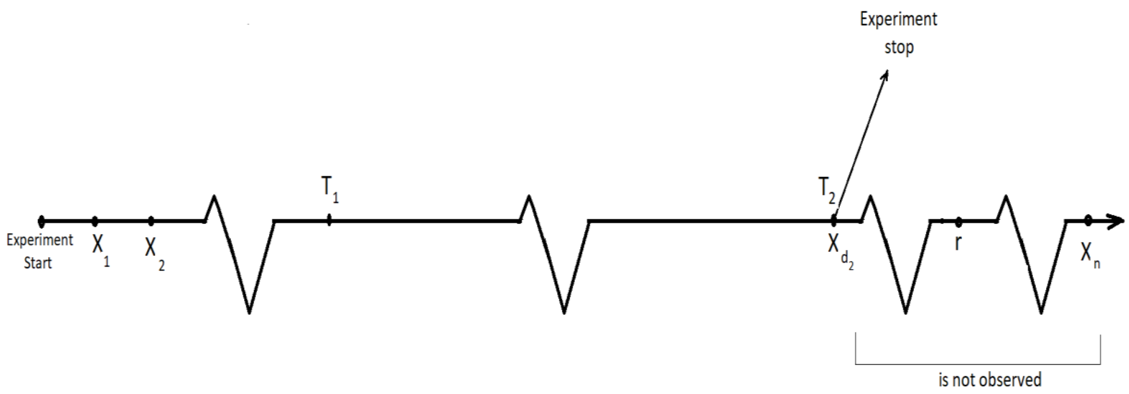


Fig. 1. Schematic representation of the G-Type-II HCS

4 Maximum Likelihood Estimation

Assume that the lifetimes of the experimental units are i.i.d. inverse Weibull random variables with cdf (2) and pdf (3). If d_1 and d_2 denote the number of failures that occur by time points T_1 and T_2 respectively, then based on the three forms of the G-Type-II HCS, the likelihood functions of α and λ are given by: then the likelihood function will take one of the following forms;

Case I

$$L_I(\alpha,\lambda) = \frac{n!}{(n-d_1)!}\left(\prod_{i=1}^{d_1} \alpha\lambda^{\alpha} x_i^{-(\alpha+1)} e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{T_1}\right)^{\alpha}}\right)^{n-d_1},$$

Case II

$$L_{II}(\alpha,\lambda) = \frac{n!}{(n-r)!}\left(\prod_{i=1}^{r} \alpha\lambda^{\alpha} x_i^{-(\alpha+1)} e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{x_r}\right)^{\alpha}}\right)^{n-r},$$

Case III

$$L_{III}(\alpha,\lambda) = \frac{n!}{(n-d_2)!}\left(\prod_{i=1}^{d_2} \alpha\lambda^{\alpha} x_i^{-(\alpha+1)} e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{T_2}\right)^{\alpha}}\right)^{n-d_2}.$$

Additionally, the corresponding log likelihood functions are:

Case I

$$l_I(\alpha,\lambda) \equiv k_1 + d_1(log\alpha + \alpha log\lambda) - (\alpha+1)\sum_{i=1}^{d_1} log x_i - \sum_{i=1}^{d_1}\left(\frac{\lambda}{x_i}\right)^{\alpha} + (n-d_1)\log\left(1-e^{-\left(\frac{\lambda}{T_1}\right)^{\alpha}}\right),$$

Case II

$$l_{II}(\alpha,\lambda) \equiv k_2 + r(log\alpha + \alpha log\lambda) - (\alpha+1)\sum_{i=1}^{r} log x_i - \sum_{i=1}^{r}\left(\frac{\lambda}{x_i}\right)^{\alpha} + (n-r)\log\left(1-e^{-\left(\frac{\lambda}{x_r}\right)^{\alpha}}\right),$$

Case III

$$l_{III}(\alpha,\lambda) \equiv k_3 + d_2(log\alpha + \alpha log\lambda) - (\alpha+1)\sum_{i=1}^{d_2} log x_i - \sum_{i=1}^{d_2}\left(\frac{\lambda}{x_i}\right)^{\alpha} + (n-d_2)\log\left(1-e^{-\left(\frac{\lambda}{T_2}\right)^{\alpha}}\right),$$

where k_1, k_2, and k_3 are normalizing constants that don't depend on the parameters.

Therefore, cases I, II, and III can be combined in a single formula written as:

$$l(\alpha,\lambda) \equiv C + \ell\, log\alpha + \ell\,\alpha\log\lambda - (\alpha+1)\sum_{i=1}^{\ell}\log x_i - \sum_{i=1}^{\ell}\left(\frac{\lambda}{x_i}\right)^{\alpha} + (n-\ell)log\left(1-e^{-\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}}\right), \tag{10}$$

where $\ell = d_1, \mathcal{R} = T_1, and\ C = k_1$ for case I, $\ell = r, \mathcal{R} = x_r, and\ C = k_2$ for case II and $\ell = d_2, \mathcal{R} = T_2, and\ C = k_3$ for case III.

The corresponding log likelihood equations are:

$$\frac{d\ln l(\alpha,\lambda)}{d\alpha} \equiv \ell\left(\frac{1}{\alpha}+ln\lambda\right)-\sum_{i=1}^{\ell}\log x_i-\sum_{i=1}^{\ell}\left(\frac{\lambda}{x_i}\right)^{\alpha} log\left(\frac{\lambda}{x_i}\right)+(n-\ell)\frac{e^{-\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}}\right)}\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha} log\left(\frac{\lambda}{\mathcal{R}}\right)=0,$$

And

$$\frac{d\ln l(\alpha,\lambda)}{d\lambda} \equiv \frac{\alpha}{\lambda}\left(\ell-\sum_{i=1}^{\ell}\left(\frac{\lambda}{x_i}\right)^{\alpha}-(n-\ell)\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}\frac{e^{-\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}}}{\left(1-e^{-\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}}\right)}\right)=0.$$

These equations cannot be solved analytically and we solve them numerically to obtain the maximum likelihood estimates $\hat{\alpha}$ and $\hat{\lambda}$ of α and λ respectively.

Once we obtain the MLE $\hat{\alpha}$, and $\hat{\lambda}$, the MLE of the entropy is obtained as:

$$\widehat{H}(f) = \gamma\left(1+\frac{1}{\hat{\lambda}}\right)+\log\left(\frac{\hat{\alpha}}{\hat{\lambda}}\right)+1. \qquad (11)$$

5 Bayes Estimation

We will derive in this section, the Bayes estimator for the residual entropy of an inverse Weibull distribution. To obtain the Bayes estimator of the residual entropy, first we will define the prior distributions of the shape (α) and the scale parameters (λ), and we will obtain the joint prior distribution of α and λ. Next, we will obtain the joint density of α, λ and the random variable X. Then, we will obtain the posterior distribution of α, λ given X. Finally, we will obtain the Bayes estimates of the residual entropy.

5.1 Prior and posterior distributions

Assume that α and λ are known a priori to have joint density of the form
$\pi(\alpha,\lambda) \propto b^{a}\,\alpha^{a-1}\,e^{-b\alpha}d^{c}\lambda^{c-1}e^{-d\lambda}$.

This mean that they are independently distributed with gamma densities g(a,b) and g(c,d) respectively, with *a*, *b*, *c*, and *d* >0. In this case the joint density of the α, λ, and X is

$$\pi(\alpha,\lambda,X) \propto b^{a}\,\alpha^{a-1}\,e^{-b\alpha}d^{c}\lambda^{c-1}e^{-d\lambda}\alpha^{\ell}\lambda^{\alpha\ell}\left(\prod_{i=1}^{\ell}x_i^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}}\right)^{n-\ell}.$$

$$= \alpha^{\ell+a-1}\,\lambda^{\alpha\ell+c-1}e^{-(b\alpha+d\lambda)}\left(\prod_{i=1}^{\ell}x_i^{-(\alpha+1)}e^{-\left(\frac{\lambda}{x_i}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{\lambda}{\mathcal{R}}\right)^{\alpha}}\right)^{n-\ell}.$$

Thus, we can obtain the posterior distribution of α and λ, given X, as follows:

$$\pi(\alpha,\lambda|X) \propto \frac{\pi(\alpha,\lambda,X)}{\int_0^{\infty}\int_0^{\infty}\pi(\alpha,\lambda,X)d\alpha d\lambda}.$$

Based on the joint prior distribution (α,λ) , we will obtain the Bayes estimator ($H^*_{F_{GHC}}$) of the residual entropy. The Bayes estimate of the residual entropy under the GHCS model is

$$H^*_{F_{GHC}} = \frac{\int_0^{\infty}\int_0^{\infty}H(f,t)\,\pi(\alpha,\lambda,X)d\alpha d\lambda}{\int_0^{\infty}\int_0^{\infty}\pi(\alpha,\lambda,X)d\alpha d\lambda}. \qquad (12)$$

6 Illustrative Example

For illustrative purposes, we use a data set given by W.B. Nelson in 1972 a subset of which is reported in Lawless [17]. The data set, as explained by Lawless himself, " is the results of a life test experiment in which pattern of a type of electrical insulating fluid were subject to a constant voltage stress". The length of time (in minutes) until each unit broke down was: 0.27, 0.4, 0.69, 0.79, 2.75, 3.91, 9.88, 13.95, 15.93, 27.8, 53.24, 82.85, 89.29, 100.58, 215.1. We imagined subjected this data to G-Type-II HCS. We take case I ($T_1 = 4, T_2 = 15, \text{and } r = 5$), case II ($T_1 = 3, T_2 = 30, \text{and } r = 9$), and case III ($T_1 = 3, T_2 = 60, \text{and } r = 12$). Table 1 presents the estimation of the entropy of the G-Type-II HCS.

7 Simulation Study

Two simulation studies were carried out; the first one to assess the performance of different estimates of the entropy under GHSC II, and the second to study the performance of the estimates of the residual entropy using different values of the parameters.

7.1 Simulation study for the entropy

Different sets of values of α , λ, T_1, T_2 , and r were used to carry out the assessment. Using Inverse Weibull distribution, a generalized type II hybrid censored data can be generated as describe next. Start by generated random sample of size n from the inverse Weibull distribution and let $x_{1:n}, \dots, x_{n:n}$ be the order statistic of this sample. Now, let d_1 and d_2 are the number of failures before T_1 and T_2 respectively. If $x_{r:n} < T_1$ then we have case I and the corresponding generalized hybrid censor sample would be $(x_{1:n} < x_{2:n} < \cdots < x_{r:n} < \cdots < x_{d_1} \leq T_1)$. If $T_1 < x_{r:n} < T_2$ then we have case II and the corresponding generalized hybrid censor sample becomes $(x_{1:n} < x_{2:n} < \cdots < T_1 < \cdots < x_{r:n})$. If $x_{r:n} > T_2$ then we have case III where we stop the experiment at T_2, and the corresponding generalized hybrid censor sample become $(x_{1:n} < x_{2:n} < \cdots < T_1 < \dots < x_{d_2} \leq T_2)$. In each case the process is replicated 10,000 times. The associated ML estimates are computed and the ML estimates of the entropy are derived. Finally, different schemes are taken into consideration to compute the relative absolute bias, relative root mean square error (RRMSE) of all estimates, and these values are tabulated in Table (2). We note the following from Table 1.

- The relative absolute bias (Rbias) and relative root mean square error (RRMSE) values of ML estimates of $\widehat{H}(X)$ at $\alpha = 9, \text{and } \lambda = 3$ has the smallest value among other value use.
- The Rbias and RRMSE values of ML estimates of $\hat{\alpha}$ at $\alpha = 10, \text{and } \lambda = 2$ has the smallest value compared to the RBias and RRMSE of ML estimates for the corresponding other sets of parameters.
- The Rbias and RRMSE values of ML estimates of $\hat{\lambda}$ at $\lambda = 3, \text{and } \alpha = 11$ has the smallest value compared to the RBias and RRMSE of ML estimates for the corresponding other sets of parameters.
- For a fixed , the RBias values increase generally as the shape parameter α increase.
- In general, for a fixed $\alpha, \lambda, n,$ and T_1 the RBias values of $\widehat{H}(X)$ increase as the stopping time point T_2 increases.
- The RBias and RRMES values of $\widehat{H}(X)$ decrease as the sample size n increase.

7.2 Simulation study for residual entropy

In this section, we assess the performance of the estimates of the residual entropy that are obtained using simulated data under GHCS Type II. The simulation encompassed different sample sizes, parameter values of the inverse Weibull distribution, and time point T_2, using the same T_1 for all. In each case, we replicate the process 1000 times. Using Equation (12), all Bayes estimates are computed with respect to the prior distribution using the *Mathematica* ® 12 software for evaluating the integration for numerator and denominator numerically. For the hyperparameters of the prior distribution the values $a = b = c = d = 1$ were used. Bayes estimates of residual entropy are derived with respect to the squared error loss (SEL) function. Finally, different schemes have been taken into consideration to compute the relative absolute bias (RBias), and relative root mean square error (RRMSE) values of all estimates and these values are tabulated in Table (3). We present the following discussions based on RBias and RRMSE;

- The RBias and RRMSE values of the residual entropy estimates ($H^{*}_{F_{GHC}}$) at $\alpha = \lambda = 2$ have the smallest values among other values use.
- For a fixed $\alpha, \lambda, r, n,$ and T_1, it seems that the RBias values increase as the stopping time T_2 increase.
- In most times, for a fixed $\alpha, \lambda, r, n,$ and T_1, it seems that the RRMSE values decrease as the stopping time T_2 increase.
- For a fixed λ the RBias values increase in general as the shape parameter α increase.
- For a fixed α the RBias values increase in general as the scale parameter λ increase.
- The RBias and RRMES values of $H^{*}_{F_{GHC}}$ become samlller as sample size n increase.

8 Summary

In this article, we derived the entropy estimators for inverse Weibull distribution using ML estimation from generalized type II hybrid censored samples. Also, simulation studies were carried out to assess the effect of different choices of censoring parameters ($n, T_1,\ T_2$ and r) of the estimates of entropy. Furthermore, we derived the residual entropy function of the inverse Weibull distribution based on generalized type II hybrid censored samples. Again, simulation studies were carried out to study the performance of the estimates of the residual entropy using different values of the censoring parameters. while we focused on the estimation of the entropy and residual entropy of the inverse Weibull distribution, the estimation of the entropy and the residual entropy functions from other distribution is the subject of a forthcoming paper.

Table 1. Estimation of entropy as an example

	T_1	T_2	r	$\hat{H}$	RBiase $\hat{H}$	MSE $\hat{H}$	RRMSE $\hat{H}$
CaseI	4	15	5	-2.24984	0.30465	0.01840	0.07866
CaseII	3	30	9	-2.29531	0.23630	0.01283	0.06101
CaseIII	3	60	12	-2.34251	0.22941	0.01273	0.05923

Table 2. Entropy estimates and relative root MSEs for $\hat{\alpha}$, $\hat{\lambda}$, and $\hat{H}$ for selected values of $\alpha, \lambda, r = 50$, $T_1 = 7$ and T_2.

λ	α	n	T_2	RBias $\hat{H}$	RRMSE $\hat{H}$	RBias α	RRMSE $\hat{\alpha}$	RBias λ	RRMSE $\hat{\lambda}$
2	8	200	10	0.017000	0.000170	0.006112	0.000061	0.000341	0.0000034
			11	0.020323	0.000203	0.006820	0.000068	0.000248	0.0000024
			12	0.024865	0.000249	0.007856	0.000079	0.000116	0.0000011
			13	0.020823	0.000208	0.006989	0.000070	0.000291	0.0000029
		150	10	0.028254	0.000283	0.009521	0.000095	0.000375	0.0000037
			11	0.027108	0.000271	0.009161	0.000092	0.000386	0.0000039
			12	0.028659	0.000287	0.009459	0.000095	0.000279	0.0000027
			13	0.032927	0.000329	0.010635	0.000106	0.000294	0.0000029
		100	10	0.039407	0.000394	0.013614	0.000136	0.000589	0.0000059
			11	0.043453	0.000435	0.014522	0.000145	0.000479	0.0000048
			12	0.041407	0.000414	0.014087	0.000141	0.000576	0.0000057
			13	0.046863	0.000469	0.015118	0.000151	0.000291	0.0000029
	9	200	10	0.033728	0.000337	0.006249	0.000062	0.000285	0.0000028
			11	0.038436	0.000384	0.006660	0.000067	0.000094	0.0000000
			12	0.031290	0.000313	0.006070	0.000061	0.000460	0.0000046
			13	0.043495	0.000435	0.007507	0.000075	0.000254	0.0000025
		150	10	0.055771	0.000558	0.009616	0.000096	0.000225	0.0000022
			11	0.052818	0.000528	0.009432	0.000094	0.000462	0.0000046
			12	0.045529	0.000455	0.008311	0.000083	0.000312	0.0000031
			13	0.130440	0.000498	0.008874	0.000089	0.000297	0.0000029
		100	10	0.082880	0.000829	0.014479	0.000145	0.000401	0.0000040
			11	10.95300	0.109530	0.014495	0.000145	3.565230	0.0356523

λ	α	n	T_2	RBias $\hat{H}$	RRMSE $\hat{H}$	RBias α	RRMSE $\hat{\alpha}$	RBias λ	RRMSE $\hat{\lambda}$
			12	10.95300	0.109536	0.013107	0.000131	3.558980	0.0355898
			13	10.95200	0.109520	0.015009	0.000150	3.567500	0.0356754
	10	200	10	0.208273	0.002083	0.006840	0.000068	0.000237	0.0000023
			11	0.190718	0.001907	0.006386	0.000064	0.000209	0.0000020
			12	0.193213	0.001932	0.006359	0.000064	0.000091	0.0000000
			13	0.189880	0.001899	0.006411	0.000064	0.000275	0.0000027
		150	10	0.278510	0.002785	0.009217	0.000092	0.000306	0.0000031
			11	0.296618	0.002966	0.009640	0.000096	0.000291	0.0000029
			12	0.275783	0.002758	0.008994	0.000090	0.000109	0.0000011
			13	0.281380	0.002814	0.009346	0.000093	0.000380	0.0000038
		100	10	0.425896	0.004259	0.014185	0.000142	0.000555	0.0000056
			11	0.404701	0.004047	0.013503	0.000135	0.000536	0.0000054
			12	0.454614	0.004546	0.014602	0.000146	0.000353	0.0000035
			13	0.390345	0.003903	0.013189	0.000132	0.000482	0.0000048
	11	200	10	0.076735	0.000767	0.007317	0.000073	0.000216	0.0000022
			11	0.061274	0.000613	0.006283	0.000063	0.000278	0.0000028
			12	0.083022	0.000830	0.007690	0.000077	0.000167	0.0000017
			13	0.081228	0.000812	0.007708	0.000077	0.000285	0.0000029
		150	10	0.093312	0.000933	0.008906	0.000089	0.000140	0.0000014
			11	0.108285	0.001083	0.010045	0.000100	0.000159	0.0000016
			12	0.103150	0.001032	0.009654	0.000097	0.000072	0.0000000
			13	0.094844	0.000948	0.009132	0.000091	0.000210	0.0000021
		100	10	0.148410	0.001484	0.014488	0.000145	0.000533	0.0000053
			11	0.151440	0.001514	0.014419	0.000144	0.000333	0.0000033
			12	0.141503	0.001415	0.013830	0.000138	0.000489	0.0000049
			13	0.151267	0.001513	0.014475	0.000145	0.000371	0.0000037
3	8	200	10	1.466000	0.014660	0.007931	0.000079	1.687800	0.0168780
			11	1.466600	0.014666	0.007806	0.000078	1.687400	0.0168740
			12	1.466600	0.014666	0.007806	0.000078	1.687400	0.0168740
			13	0.009320	0.000093	0.007347	0.000073	0.000068	0.0000000
		150	10	0.011011	0.000110	0.009221	0.000092	0.000162	0.0000016
			11	0.012852	0.000129	0.010054	0.000101	0.000189	0.0000019
			12	0.013829	0.000138	0.010838	0.000108	0.000030	0.0000000
			13	0.013783	0.000138	0.010732	0.000107	0.000068	0.0000000
		100	10	0.018095	0.000181	0.014830	0.000148	0.000279	0.0000028
			11	0.017844	0.000178	0.014480	0.000145	0.000007	0.0000000
			12	0.017319	0.000173	0.014541	0.000145	0.000444	0.0000044
			13	0.018284	0.000183	0.014810	0.000148	0.000111	0.0000011
	9	200	10	0.011150	0.000112	0.007285	0.000073	0.000045	0.0000000
			11	0.012255	0.000123	0.007942	0.000079	0.000045	0.0000000
			12	0.010871	0.000109	0.007424	0.000074	0.000250	0.0000025
			13	0.008610	0.000086	0.006255	0.000063	0.000235	0.0000024
		150	10	0.012394	0.000124	0.008886	0.000089	0.000503	0.0000050
			11	0.015300	0.000154	0.010201	0.000102	0.000179	0.0000018
			12	0.017135	0.000171	0.011118	0.000111	0.000196	0.0000020
			13	0.014070	0.000141	0.009530	0.000095	0.000205	0.0000021
		100	10	0.019491	0.000194	0.013556	0.000136	0.000357	0.0000036
			11	0.019661	0.000197	0.013608	0.000136	0.000351	0.0000035
			12	0.022523	0.000225	0.015173	0.000152	0.000378	0.0000038
			13	0.019950	0.000200	0.013885	0.000139	0.000486	0.0000049
	10	200	10	0.012495	0.000125	0.007037	0.000070	0.000338	0.0000034
			11	0.014010	0.000140	0.007479	0.000075	0.000154	0.0000015
			12	0.011203	0.000112	0.006380	0.000064	0.000252	0.0000025
			13	0.011818	0.000118	0.006658	0.000067	0.000265	0.0000027
		150	10	0.017985	0.000180	0.009766	0.000098	0.000230	0.0000023

λ	α	n	T_2	RBias $\widehat{H}$	RRMSE $\widehat{H}$	RBias α	RRMSE $\widehat{\alpha}$	RBias λ	RRMSE $\hat{\lambda}$
			11	0.017619	0.000176	0.009591	0.000096	0.000318	0.0000032
			12	0.014677	0.000147	0.008549	0.000085	0.000401	0.0000040
			13	0.016022	0.000160	0.008986	0.000090	0.000338	0.0000034
		100	10	0.025386	0.000254	0.014017	0.000140	0.000386	0.0000039
			11	0.024162	0.000242	0.013544	0.000135	0.000447	0.0000045
			12	0.025809	0.000258	0.014229	0.000142	0.000356	0.0000036
			13	0.025559	0.000256	0.014084	0.000141	0.000360	0.0000036
	11	200	10	0.013288	0.000133	0.007204	0.000072	0.000157	0.0000016
			11	0.013189	0.000132	0.007110	0.000071	0.000129	0.0000013
			12	0.012310	0.000123	0.006714	0.000067	0.000084	0.0000000
			13	0.013685	0.000137	0.007381	0.000074	0.000172	0.0000017
		150	10	0.016727	0.000167	0.009107	0.000091	0.000148	0.0000015
			11	0.018324	0.000183	0.009907	0.000099	0.000240	0.0000024
			12	0.016584	0.000166	0.009191	0.000092	0.000271	0.0000027
			13	0.016723	0.000167	0.009180	0.000092	0.000207	0.0000021
		100	10	0.026002	0.000260	0.014317	0.000143	0.000506	0.0000051
			11	0.034429	0.000344	0.014472	0.000145	0.000394	0.0000039
			12	0.032598	0.000326	0.013882	0.000139	0.000330	0.0000033
			13	0.035905	0.000359	0.014998	0.000150	0.000381	0.0000038

Table 3. The residual entropy estimates of H^*_{FGHC} and its relative bias and relative root MSEs for selected values of α, λ, $r = 50$, $T_1 = 7$ and T_2, when $a = b = c = d = 1$ and $t = 2$

λ	α	n	T_2	H^*_{FGHC}	RBias	RRMSE
1	1	200	10	5.5252	0.008199	0.000082
			11	5.5252	0.008193	0.000082
			12	5.5253	0.008215	0.000082
			13	5.5254	0.008228	0.000082
		150	10	5.5415	0.011170	0.000112
			11	5.5414	0.011143	0.000111
			12	5.5439	0.011612	0.000116
			13	5.5377	0.010483	0.000105
		100	10	5.5705	0.016461	0.000165
			11	5.5697	0.016316	0.000163
			12	5.5691	0.016199	0.000162
			13	5.5708	0.016519	0.000165
2	2	200	10	4.3023	0.000338	0.000011
			11	4.3033	0.000579	0.000018
			12	4.3056	0.001109	0.000035
			13	4.2977	0.000721	0.000023
		150	10	4.3045	0.000855	0.000027
			11	4.2995	0.000312	0.000010
			12	4.3035	0.000613	0.000019
			13	4.2992	0.000374	0.000012
		100	10	4.3108	0.002315	0.000073
			11	4.3065	0.001322	0.000042
			12	4.2952	0.001314	0.000042
			13	4.3062	0.001256	0.000040
3	3	200	10	5.2222	0.252400	0.007982
			11	5.2355	0.250499	0.007921
			12	5.2413	0.249673	0.007895
			13	5.2309	0.251154	0.007942
		150	10	6.2439	0.106136	0.003356
			11	6.2337	0.107600	0.003403

λ	α	n	T_2	H^*_{FGHC}	RBias	RRMSE
			12	6.2403	0.106658	0.003373
			13	6.2406	0.106606	0.003371
		100	10	6.7429	0.034703	0.001097
			11	6.7642	0.031652	0.001001
			12	6.7502	0.033663	0.001065
			13	6.7608	0.032146	0.001017
3	9	200	10	8.1356	0.296100	0.002961
			11	8.1471	0.295100	0.002951
			12	8.1364	0.296000	0.002960
			13	8.1377	0.295900	0.002950
		150	10	8.3069	0.281300	0.002813
			11	8.2957	0.282200	0.002820
			12	8.3220	0.280000	0.002800
			13	8.3209	0.280100	0.002800
		100	10	8.8433	0.234900	0.002300
			11	8.7873	0.239700	0.002398
			12	8.8289	0.236100	0.002362
			13	8.8235	0.236600	0.002366
0.5	4	200	10	6.5859	0.022931	0.000725
			11	6.5815	0.023586	0.000746
			12	6.5908	0.022206	0.000702
			13	6.5912	0.022151	0.000700
		150	10	6.5856	0.022976	0.000727
			11	6.5587	0.026963	0.000853
			12	6.5691	0.025428	0.000804
			13	6.5681	0.025577	0.000809
		100	10	6.5444	0.029074	0.000919
			11	6.5475	0.028625	0.000905
			12	6.5445	0.029076	0.000919
			13	6.5418	0.029468	0.000932
0.5	5	200	10	6.5523	0.047029	0.001487
			11	6.5564	0.046441	0.001469
			12	6.5465	0.047871	0.001514
			13	6.5483	0.047612	0.001506
		150	10	6.5357	0.049446	0.001564
			11	6.5281	0.050546	0.001598
			12	6.5401	0.048807	0.001543
			13	6.5387	0.049015	0.001550
		100	10	6.5039	0.054077	0.001710
			11	6.5108	0.053063	0.001678
			12	6.5018	0.054383	0.001720
			13	6.5073	0.053584	0.001695
0.5	6	200	10	6.5049	0.072039	0.002278
			11	6.5110	0.071169	0.002250
			12	6.4997	0.072778	0.002301
			13	6.4978	0.073059	0.002310
		150	10	6.4969	0.073176	0.002314
			11	6.4955	0.073381	0.002320
			12	6.4881	0.074442	0.002354
			13	6.4898	0.074190	0.002346
		100	10	6.4922	0.073848	0.002335
			11	6.4814	0.075395	0.002384
			12	6.4797	0.075633	0.002391
			13	6.4861	0.074726	0.002363
4	9	200	10	12.472	0.090100	0.000901

λ	α	n	T_2	H^*_{FGHC}	RBias	RRMSE
			11	12.452	0.091580	0.000916
			12	12.493	0.088580	0.000886
			13	12.458	0.091150	0.000912
		150	10	13.543	0.011961	0.000120
			11	13.522	0.013521	0.000135
			12	13.580	0.009294	0.000093
			13	13.466	0.017560	0.000176
		100	10	16.577	0.209346	0.002093
			11	16.650	0.214719	0.002147
			12	16.372	0.194396	0.001944
			13	16.324	0.190929	0.001909
0.5	3	200	10	6.5822	0.005511	0.000174
			11	6.5864	0.004878	0.000154
			12	6.5869	0.004812	0.000152
			13	6.6058	0.001956	0.000062
		150	10	6.5938	0.003766	0.000119
			11	6.5760	0.006453	0.000204
			12	6.5945	0.003657	0.000116
			13	6.5938	0.003759	0.000119
		100	10	6.5754	0.006543	0.000207
			11	6.5748	0.006631	0.000210
			12	6.5423	0.011543	0.000365
			13	6.5739	0.006766	0.000214
1	5	200	10	5.9101	0.066481	0.002102
			11	5.9100	0.066486	0.002103
			12	5.9093	0.066604	0.002106
			13	5.9075	0.066890	0.002115
		150	10	5.9011	0.067894	0.002147
			11	5.9009	0.067921	0.002148
			12	5.9032	0.067557	0.002136
			13	5.9007	0.067952	0.002149
		100	10	5.8867	0.070172	0.002219
			11	5.8869	0.070131	0.002218
			12	5.8909	0.069502	0.002198
			13	5.8912	0.069456	0.002196

9 Conclusion

Two simulation studies were carried out; in the first one, we obtained the entropy estimates and its RBiase and RRMSE. In the second one, we obtained the residual entropy estimates and its RBiase and RRMSE. From the two studies the results show that the estimates in general is very robust against changes of n, T_1, T_2 and r resulting in low levels of RBiase and RRMSE. These results are valid for reasonably small initial sample sizes.

Competing Interests

Author has declared that no competing interests exist.

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