

Research Article

Study of $b \longrightarrow c$ Induced $\bar{B}^* \longrightarrow V\ell\bar{\nu}_\ell$ Decays

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In this paper, we investigate the tree-dominated $\bar{B}_{u,d,s,c}^* \longrightarrow V\ell\bar{\nu}_\ell$ ($V = D_{u,d}^*, D_s^*, J/\psi$ and $\ell = e, \mu, \tau$) decays in the Standard Model with the relevant form factors obtained in the light-front quark model. These decays involve much more helicity states relative to the corresponding $\bar{B}^* \longrightarrow P\ell\bar{\nu}_\ell$ and $\bar{B} \longrightarrow V\ell\bar{\nu}_\ell$ decays, and moreover, the contribution of longitudinal polarization mode (V meson) is relatively small, $\sim 30\%$, compared with the corresponding B meson decays. We have also computed the branching fraction, lepton spin asymmetry, forward-backward asymmetry, and ratio $R_V^{*(L)} \equiv \mathcal{B}(\bar{B}^* \longrightarrow V\tau\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^* \longrightarrow V\ell\bar{\nu}_\ell)$ ($\ell' = e, \mu$). Numerically, the branching fractions of $\bar{B}^* \longrightarrow V\ell\bar{\nu}_\ell$ decays are at the level of $O(10^{-7})$ and are hopeful to be observed by LHC and Belle-II experiments. The ratios $R_{D_s^*, D_s^*, J/\psi}^{*(L)}$ have relatively small theoretical uncertainties and are close to each other, $R_{D_s^*}^{*(L)} \approx R_{D_s^*}^{*(L)} \approx R_{J/\psi}^{*(L)} \approx [0.26, 0.27]([0.27, 0.29])$, which are a bit different from the predictions in some previous works. The future measurements are expected to make tests on these predictions.

1. Introduction

In the past years, a large amount of $B\bar{B}$ events have been accumulated by Babar, Belle, Tevatron and LHCb experiments, and most of B -meson decays having branching fractions $\geq O(10^{-7})$ have been measured [1]. Moreover, some deviations between the standard model (SM) predictions and the experimental data have been observed, for instance, the angular observable P_5' of $B \longrightarrow K^*\mu^+\mu^-$ decay with 2.6σ discrepancy [2–6], the differential branching fraction of $B_s \longrightarrow \phi\mu^+\mu^-$ decay with 3.3σ discrepancy [7, 8], and the well-known “ πK CP puzzle” [9, 10]. Besides the flavor-changing-neutral-current processes mentioned above, the B -meson semileptonic decays induced by $b \longrightarrow c\ell\bar{\nu}_\ell$ transition also play an important role in testing the SM and probing the hints of possible new physics (NP). For instance, the well-known “ R_{D^*} anomaly” reported by BaBar [11, 12], Belle [13–15], and LHCb [16, 17] collaborations exhibits a significant deviation between the SM prediction and experimental data [1, 18, 19]. Many studies have been done within the model-independent frameworks [20–27], as well as in some specific

NP models, for instance Refs. [28–48]. One can refer to Refs. [49, 50] for recent reviews.

The spin-triplet vector B_q^* meson with a quantum number of $n^{2s+1}L_J = 1^3S_1$ and $J^P = 1^-$ [51–54] has the same flavor components as the spin-singlet pseudoscalar B_q ($q = u, d, s$ and c) meson and can also decay through the $b \longrightarrow c\ell\bar{\nu}_\ell$ transition at quark-level; therefore, its $b \longrightarrow c$ -induced semileptonic decays can play a similar role as B meson decays for testing the SM and probing possible hints of NP.

The B_q^* meson is an unstable particle, it cannot decay via strong interaction due to that $m_{B_q^*} - m_{B_q} \leq 50 \text{ MeV} < m_\pi$ [55]; B_q^* meson decay is dominated by the radiative process [55], $B_q^* \longrightarrow B_q\gamma$; the weak decay modes via the bottom-changing transition (for instance, the $b \longrightarrow c$ induced semileptonic B_q^* decays considered in this work) are generally very rare, and their branching fractions are expected to be very small within the SM. Until now, there is no experimental information and few theoretical works concentrating on the B_q^* weak decays. Fortunately, thanks to the high luminosity and large production cross section at the running LHC

and SuperKEKB/Belle-II experiments, a huge amount of the B_q^* meson data samples would be accumulated. At Belle-II experiment, the B^* and B_s^* mesons are produced mainly via $Y(5S)$ decays. With the target annual integrated luminosity, $\sim 13ab^{-1}$ [56], and the cross section of $Y(5S)$ production in e^+e^- collisions, $\sigma(e^+e^- \rightarrow Y(5S)) = (0.301 \pm 0.002 \pm 0.039)$ nb [57], it is expected that about 4×10^9 $Y(5S)$ samples could be produced per year by Belle-II. Further considering that $Y(5S)$ meson mainly decays to final states with a pair of $B_{(s)}^{(*)}$ mesons and using the branching fractions of $Y(5S)$ decays given by PDG [55], it can be estimated that about $N(B^* + \bar{B}^*)/year \sim 4 \times 10^9$ and $N(B_s^* + \bar{B}_s^*)/year \sim 2 \times 10^9$ samples can be accumulated by Belle-II per year. Unfortunately, the B_c^* meson and its decays are out of the scope of Belle-II experiment. In addition, a lot of B_q^* samples can also be produced via pp collision and be accumulated in the future by LHC with high collision energy, high luminosity and rather large production cross section [58–60], and some B_q^* weak decays are hopeful to be observed, such as the leptonic $B_s^* \rightarrow \ell^+\ell^-$ decay with branching fraction $\sim O(10^{-11})$ [61].

Encouraged by the abundant B_q^* data samples at future heavy-flavor experiments, some interesting theoretical studies for the B_q^* weak decays have been made within the SM, for instance, the pure leptonic $\bar{B}_s^* \rightarrow \ell^+\ell^-$ and $\bar{B}_{u,c}^* \rightarrow \ell^-\bar{\nu}_\ell$ decays [61, 62], the impact of $\bar{B}_{s,d}^* \rightarrow \mu^+\mu^-$ on $\bar{B}_{s,d} \rightarrow \mu^+\mu^-$ decays [63], the studies of the semileptonic B_c^* decays within the QCD sum rules [64–66], the semileptonic $B_{u,d,c,s}^* \rightarrow (P, V)\ell^-\bar{\nu}_\ell$ with $P = D, D_s, \eta_c, V = D^*, D_s^*, J/\psi$ decays within the Bethe-Salpeter (BS) method [67], and an approach under the assumption of heavy quark symmetry (HQS) [68], $\bar{B}^* \rightarrow P\ell^-\bar{\nu}_\ell$ with $P = D, D_s, \pi, K$ [69] and the nonleptonic $\bar{B}_{d,s}^{*0} \rightarrow D_{d,s}^+M^-$ ($M = \pi, K, \rho$ and K^*) [70, 71], $\bar{B}_{d,s}^* \rightarrow D_{d,s}V$ [72], $B_c^* \rightarrow B_{u,d,s}V, B_{u,d,s}P$ [73], $B_c^* \rightarrow \eta_c V$ [74], $B^* \rightarrow \bar{D}D$ [75], and $B_c^* \rightarrow \psi(1S, 2S)P, \eta_c(1S, 2S)P$ [76] decays. Moreover, the NP effects on the semileptonic $\bar{B}^* \rightarrow P\ell^-\bar{\nu}_\ell$ with $P = D, D_s, \pi, K$ decays have been investigated in a model-independent scheme [77] and the vector leptoquark model [78]. In this paper, we pay our attention to the CKM-favored and tree-dominated semileptonic $\bar{B}_{u,d,s,c}^* \rightarrow V\ell\bar{\nu}_\ell$ ($V = D_{u,d}^*, D_s^*, J/\psi$) weak decays, which are generally much more complicated than the corresponding B decay modes because they involve much more allowed helicity states.

Our paper is organized as follows. In Section 2, the helicity amplitudes and observables of $\bar{B}^* \rightarrow V\ell\bar{\nu}_\ell$ decays are calculated. Section 3 is devoted to the numerical results and discussions, and the $\bar{B}^* \rightarrow V$ transition form factors obtained within the covariant light-front quark model are used in the computation. Finally, we give our summary in Section 4.

2. Theoretical Framework and Results

2.1. Effective Lagrangian and Amplitude. In the SM, $\bar{B}_{u,d,s,c}^* \rightarrow V\ell\bar{\nu}_\ell$ ($V = D_{u,d}^*, D_s^*, J/\psi$) decays are induced by $b \rightarrow c$

$\ell\bar{\nu}_\ell$ transition at quark level via W -exchange and can be described by the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -2\sqrt{2}G_F V_{cb} \bar{c}_L \gamma^\mu b_L \bar{\ell}_L \gamma_\mu \nu_L + \text{h.c.}, \quad (1)$$

at low energy scale $\mu = O(m_b)$, where G_F is the Fermi coupling constant and V_{cb} denotes the CKM matrix element. Using Eq. (1), the amplitude of $\bar{B}^* \rightarrow V\ell\bar{\nu}_\ell$ decay can be written as the product of the hadronic matrix element and leptonic current. Then, in terms of leptonic ($L_{\mu\nu}$) and hadronic ($H^{\mu\nu}$) tensors built from the respective products of the leptonic and hadronic currents, the square amplitude can be expressed as

$$|M(\bar{B}^* \rightarrow V\ell^-\bar{\nu}_\ell)|^2 = |\langle V\ell^-\bar{\nu}_\ell | \mathcal{L}_{\text{eff}} | \bar{B}^* \rangle|^2 = \frac{G_F^2 |V_{cb}|^2}{2} L_{\mu\nu} H^{\mu\nu}. \quad (2)$$

Inserting the completeness relation of the polarization vector of virtual W^* boson,

$$\sum_{m,n} \bar{\epsilon}_\mu(m) \bar{\epsilon}_\nu^*(n) g_{mn} = g_{\mu\nu}, \quad (3)$$

the product of $L_{\mu\nu}$ and $H^{\mu\nu}$ can be rewritten as

$$L_{\mu\nu} H^{\mu\nu} = \sum_{m,m',n,n'} L(m,n) H(m',n') g_{mm'} g_{nn'}, \quad (4)$$

where $L(m,n) \equiv L^{\mu\nu} \bar{\epsilon}_\mu(m) \bar{\epsilon}_\nu^*(n)$ and $H(m,n) \equiv H^{\mu\nu} \bar{\epsilon}_\mu(m) \bar{\epsilon}_\nu^*(n)$ are Lorentz invariant and therefore can be evaluated in different reference frames. In our following evaluation, $H(m,n)$ and $L(m,n)$ will be calculated in the B^* -meson rest frame and the $\ell - \bar{\nu}_\ell$ center-of-mass frame, respectively.

2.2. Kinematics. In the rest frame of B^* meson, assuming the final state V -meson moving along with positive z -direction, the momenta of B^* , V , and W^* could be written as

$$\begin{aligned} \vec{p}_{B^*}^\mu &= (m_{B^*}, 0, 0, 0), \\ \vec{p}_V^\mu &= \left(E_V, 0, 0, \left| \vec{p} \right| \right), \\ \vec{q}^\mu &= \left(q^0, 0, 0, -\left| \vec{p} \right| \right), \end{aligned} \quad (5)$$

respectively, where $q^0 = (m_{B^*}^2 - m_V^2 + q^2)/2m_{B^*}$ and $|\vec{p}| = \lambda^{1/2}(m_{B^*}^2, m_V^2, q^2)/2m_{B^*}$, with $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2(ab + bc + ca)$ and $q^2 = (p_{B^*} - p_V)^2$ being the momentum transfer squared, are the energy and momentum of virtual W^* . The polarization vectors of the initial B^* -meson and daughter V -meson, $\epsilon_1^\mu(0, \pm)$ and $\epsilon_2^\mu(0, \pm)$, can be written as

$$\begin{aligned}
\varepsilon_1^\mu(0) &= (0, 0, 0, 1), \\
\varepsilon_1^\mu(\pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0); \\
\varepsilon_2^\mu(0) &= \frac{1}{m_V} \left(\left| \vec{p} \right|, 0, 0, E_V \right), \\
\varepsilon_2^\mu(\pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0),
\end{aligned} \tag{6}$$

respectively. For the four polarization vectors of virtual W^* , $\bar{\varepsilon}^\mu(t, 0, \pm)$, one can conveniently choose [79, 80]

$$\begin{aligned}
\bar{\varepsilon}^\mu(t) &= \frac{1}{\sqrt{q^2}} \left(q^0, 0, 0, -\left| \vec{p} \right| \right), \\
\bar{\varepsilon}^\mu(0) &= \frac{1}{\sqrt{q^2}} \left(\left| \vec{p} \right|, 0, 0, -q^0 \right), \\
\bar{\varepsilon}^\mu(\pm) &= \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0),
\end{aligned} \tag{7}$$

in which, $\lambda_{W^*} = t$ has to be understood as $\lambda_{W^*} = 0$ and $J = 0$.

Turning to the $\ell - \bar{\nu}_\ell$ center-of-mass frame, the four-momenta of lepton and antineutrino are given as

$$\begin{aligned}
p_\ell^\mu &= \left(E_\ell, \left| \vec{p}_\ell \right| \sin \theta, 0, \left| \vec{p}_\ell \right| \cos \theta \right), \\
p_{\bar{\nu}_\ell}^\mu &= \left(\left| \vec{p}_\ell \right|, -\left| \vec{p}_\ell \right| \sin \theta, 0, -\left| \vec{p}_\ell \right| \cos \theta \right),
\end{aligned} \tag{8}$$

where $E_\ell = (q^2 + m_\ell^2)/2\sqrt{q^2}$, $\left| \vec{p}_\ell \right| = (q^2 - m_\ell^2)/2\sqrt{q^2}$, and θ is the angle between V and ℓ three-momenta. In this frame, the polarization vectors $\bar{\varepsilon}^\mu(\lambda_{W^*})$ have the form

$$\bar{\varepsilon}^\mu(t) = (1, 0, 0, 0), \tag{9}$$

$$\bar{\varepsilon}^\mu(0) = (0, 0, 0, 1), \tag{10}$$

$$\bar{\varepsilon}^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \tag{11}$$

2.3. Hadronic Helicity Amplitudes. For hadronic part, one has to calculate the hadronic helicity amplitudes $H_{\lambda_{W^*} \lambda_{B^*} \lambda_V}$ of $\bar{B}^* \rightarrow V \ell^- \bar{\nu}_\ell$ decay defined by

$$\begin{aligned}
H_{\lambda_{W^*} \lambda_{B^*} \lambda_V}(q^2) &= \left\langle V(p_V, \lambda_V) \mid \bar{c} \gamma_\mu (1 - \gamma_5) b \mid \bar{B}^*(p_{B^*}, \lambda_{B^*}) \right\rangle \bar{\varepsilon}^{*\mu}(\lambda_{W^*}),
\end{aligned} \tag{12}$$

which describes the decay of three helicity states of B^* meson into the three helicity states of daughter V meson and the four helicity states of virtual W^* . For the $B^* \rightarrow V$ transition, the matrix elements $\langle V(p_V, \lambda_V) \mid \bar{c} \gamma_\mu (1 - \gamma_5) b \mid \bar{B}^*(p_{B^*}, \lambda_{B^*}) \rangle$ can be factorized in terms of ten form factors $V_{1,2,3,4,5,6}(q^2)$ and $A_{1,2,3,4}(q^2)$ as [81, 82]

$$\begin{aligned}
&\left\langle V(\varepsilon_2, p_V) \mid \bar{c} \gamma_\mu b \mid \bar{B}^*(\varepsilon_1, p_{B^*}) \right\rangle \\
&= (\varepsilon_1 \cdot \varepsilon_2^*) \left[-P_\mu V_1(q^2) + q_\mu V_2(q^2) \right] \\
&\quad + \frac{(\varepsilon_1 \cdot q)(\varepsilon_2^* \cdot q)}{m_{B^*}^2 - m_V^2} \left[P_\mu V_3(q^2) - q_\mu V_4(q^2) \right] \\
&\quad - (\varepsilon_1 \cdot q) \varepsilon_{2\mu}^* V_5(q^2) + (\varepsilon_2^* \cdot q) \varepsilon_{1\mu} V_6(q^2), \\
&\left\langle V(\varepsilon_2, p_V) \mid \bar{c} \gamma_5 \gamma_\mu b \mid \bar{B}^*(\varepsilon_1, p_{B^*}) \right\rangle \\
&= -i \varepsilon_{\mu\nu\alpha\beta} \varepsilon_1^\alpha \varepsilon_2^{*\beta} \left[P^\nu A_1(q^2) - q^\nu A_2(q^2) \right] \\
&\quad - \frac{i \varepsilon_2^* \cdot q}{m_{B^*}^2 - m_V^2} \varepsilon_{\mu\nu\alpha\beta} \varepsilon_1^\nu P^\alpha q^\beta A_3(q^2) \\
&\quad + \frac{i \varepsilon_1 \cdot q}{m_{B^*}^2 - m_V^2} \varepsilon_{\mu\nu\alpha\beta} \varepsilon_2^{*\nu} P^\alpha q^\beta A_4(q^2)
\end{aligned} \tag{13}$$

with the sign convention $\varepsilon_{0123} = -1$.

Then, by contracting these hadronic matrix elements with the polarization vector of virtual W^* boson, we can finally obtain the nonvanishing hadronic helicity amplitudes, $H_{\lambda_{W^*} \lambda_{B^*} \lambda_V}$, given as

$$\begin{aligned}
H_{0^{++}}(q^2) &= -\frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} A_1(q^2) + \sqrt{q^2} A_2(q^2) + \frac{2m_{B^*} \left| \vec{p} \right|}{\sqrt{q^2}} V_1(q^2), \\
H_{1^{++}}(q^2) &= -\frac{2m_{B^*} \left| \vec{p} \right|}{\sqrt{q^2}} A_1(q^2) + \frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} V_1(q^2) - \sqrt{q^2} V_2(q^2), \\
H_{-+0}(q^2) &= -\frac{m_{B^*}^2 + 3m_V^2 - q^2}{2m_V} A_1(q^2) + \frac{(m_{B^*}^2 - m_V^2 - q^2)}{2m_V} A_2(q^2) \\
&\quad - \frac{2m_{B^*} \left| \vec{p} \right|^2}{m_V(m_{B^*}^2 - m_V^2)} A_3(q^2) - \frac{m_{B^*} \left| \vec{p} \right|}{m_V} V_6(q^2), \\
H_{0^{--}}(q^2) &= \frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} A_1(q^2) - \sqrt{q^2} A_2(q^2) + \frac{2m_{B^*} \left| \vec{p} \right|}{\sqrt{q^2}} V_1(q^2), \\
H_{1^{--}}(q^2) &= \frac{2m_{B^*} \left| \vec{p} \right|}{\sqrt{q^2}} A_1(q^2) + \frac{m_{B^*}^2 - m_V^2}{\sqrt{q^2}} V_1(q^2) - \sqrt{q^2} V_2(q^2), \\
H_{+ - 0}(q^2) &= \frac{m_{B^*}^2 + 3m_V^2 - q^2}{2m_V} A_1(q^2) - \frac{(m_{B^*}^2 - m_V^2 - q^2)}{2m_V} A_2(q^2) \\
&\quad + \frac{2m_{B^*} \left| \vec{p} \right|^2}{m_V(m_{B^*}^2 - m_V^2)} A_3(q^2) - \frac{m_{B^*} \left| \vec{p} \right|}{m_V} V_6(q^2), \\
H_{+ + 0}(q^2) &= \frac{3m_{B^*}^2 + m_V^2 - q^2}{2m_{B^*}} A_1(q^2) - \frac{(m_{B^*}^2 - m_V^2 + q^2)}{2m_{B^*}} A_2(q^2) \\
&\quad + \frac{2m_{B^*} \left| \vec{p} \right|^2}{m_{B^*}^2 - m_V^2} A_4(q^2) - \left| \vec{p} \right| V_5(q^2),
\end{aligned}$$

$$\begin{aligned}
H_{-0-}(q^2) &= -\frac{3m_{B^*}^2 + m_V^2 - q^2}{2m_{B^*}} A_1(q^2) + \frac{(m_{B^*}^2 - m_V^2 + q^2)}{2m_{B^*}} A_2(q^2) \\
&\quad - \frac{2m_{B^*} |\vec{p}|^2}{m_{B^*}^2 - m_V^2} A_4(q^2) - |\vec{p}| V_5(q^2), \\
H_{000}(q^2) &= \frac{|\vec{p}| (m_{B^*}^2 + m_V^2 - q^2)}{\sqrt{q^2} m_V} V_1(q^2) + \frac{2m_{B^*}^2 |\vec{p}|^3}{\sqrt{q^2} m_V (m_{B^*}^2 - m_V^2)} V_3(q^2) \\
&\quad - \frac{|\vec{p}| (m_{B^*}^2 - m_V^2 - q^2)}{2\sqrt{q^2} m_V} V_5(q^2) + \frac{|\vec{p}| (m_{B^*}^2 - m_V^2 + q^2)}{2\sqrt{q^2} m_V} V_6(q^2), \\
H_{100}(q^2) &= \frac{(m_{B^*}^2 - m_V^2)(m_{B^*}^2 + m_V^2 - q^2)}{2\sqrt{q^2} m_{B^*} m_V} V_1(q^2) \\
&\quad - \frac{\sqrt{q^2} (m_{B^*}^2 + m_V^2 - q^2)}{2m_{B^*} m_V} V_2(q^2) + \frac{m_{B^*} |\vec{p}|^2}{\sqrt{q^2} m_V} V_3(q^2) \\
&\quad - \frac{m_{B^*} |\vec{p}|^2 \sqrt{q^2}}{m_V (m_{B^*}^2 - m_V^2)} V_4(q^2) - \frac{m_{B^*} |\vec{p}|^2}{\sqrt{q^2} m_V} V_5(q^2) + \frac{m_{B^*} |\vec{p}|^2}{\sqrt{q^2} m_V} V_6(q^2).
\end{aligned} \tag{14}$$

Obviously, only the amplitudes with $\lambda_{B^*} = \lambda_V - \lambda_{W^*}$ survive due to the helicity conservation.

2.4. Helicity Amplitudes and Observables. For the leptonic part, the leptonic tensor could be expanded in terms of a complete set of Wigner's d^J -functions, which have been widely used in the study of hadron semileptonic [79, 83, 84]. As a result, $L_{\mu\nu} H^{\mu\nu}$ can be reduced to a very compact form

$$\begin{aligned}
L_{\mu\nu} H^{\mu\nu} &= \frac{1}{8} \sum_{\substack{\lambda_\ell, \lambda_{\bar{\nu}_\ell}, J \\ \lambda_{W^*}, \lambda_{W^*}}} (-1)^{J+J'} |h_{\lambda_\ell, \lambda_{\bar{\nu}_\ell}}|^2 \delta_{\lambda_{B^*}, \lambda_V - \lambda_{W^*}} \delta_{\lambda_{B^*}, \lambda_V - \lambda_{W^*}} \\
&\quad \times d_{\lambda_{W^*}, \lambda_\ell - 1/2}^J d_{\lambda_{W^*}, \lambda_\ell - 1/2}^{J'} H_{\lambda_{W^*}, \lambda_{B^*}, \lambda_V} H_{\lambda_{W^*}, \lambda_{B^*}, \lambda_V}
\end{aligned} \tag{15}$$

where J and J' run over 1 and 0, $\lambda_{W^*}^{(J)}$, and λ_ℓ run over their components. For the standard expression of d^J function, we take their value from PDG [55]. The leptonic helicity amplitude $h_{\lambda_\ell, \lambda_{\bar{\nu}_\ell}}$ in Eq. (15) defined as

$$h_{\lambda_\ell, \lambda_{\bar{\nu}_\ell}} = \bar{u}_\ell(\lambda_\ell) \gamma^\mu (1 - \gamma_5) \nu_{\bar{\nu}_\ell} \left(\frac{1}{2} \right) \bar{\epsilon}_\mu(\lambda_{W^*}), \tag{16}$$

Taking the exact forms of spinors and W^* polarization vectors given in Eq. (11), we obtain

$$\begin{aligned}
|h_{-\frac{1}{2}, 1}|^2 &= 8(q^2 - m_\ell^2), \\
|h_{1/2, 1/2}|^2 &= 8 \frac{m_\ell^2}{2q^2} (q^2 - m_\ell^2),
\end{aligned} \tag{17}$$

which are the same as the results obtained in semileptonic B and hyperon decays [83, 84].

Using the amplitudes obtained above, we can then further evaluate the observables of $\bar{B}^* \rightarrow V \ell^- \bar{\nu}_\ell$ decays. The double differential decay rate is written as

$$\frac{d^2 \Gamma}{dq^2 d \cos \theta} = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3} \frac{|\vec{p}|}{8m_{B^*}^2} \frac{1}{3} \left(1 - \frac{m_\ell^2}{q^2}\right) L_{\mu\nu} H^{\mu\nu}, \tag{18}$$

where the factor 1/3 is caused by averaging over the spins of initial \bar{B}^* meson. The double differential decay rate with a given helicity state of lepton ($\lambda_\ell = \pm 1/2$) is written as

$$\begin{aligned}
\frac{d^2 \Gamma[\lambda_\ell = -1/2]}{dq^2 d \cos \theta} &= \frac{G_F^2 |V_{cb}|^2 |\vec{p}|}{256\pi^3 m_{B^*}^2} \frac{1}{3} q^2 \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \\
&\quad \times \left[(1 - \cos \theta)^2 (H_{+0+}^2 + H_{+0-}^2) \right. \\
&\quad \left. + (1 + \cos \theta)^2 (H_{-0-}^2 + H_{-0+}^2) \right. \\
&\quad \left. + 2 \sin^2 \theta (H_{0++}^2 + H_{0--}^2 + H_{000}^2) \right],
\end{aligned} \tag{19}$$

$$\begin{aligned}
\frac{d^2 \Gamma[\lambda_\ell = 1/2]}{dq^2 d \cos \theta} &= \frac{G_F^2 |V_{cb}|^2 |\vec{p}|}{256\pi^3 m_{B^*}^2} \frac{1}{3} q^2 \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \frac{m_\ell^2}{q^2} \\
&\quad \cdot \left[\sin^2 \theta (H_{+0+}^2 + H_{+0-}^2 + H_{-0-}^2 + H_{-0+}^2) \right. \\
&\quad \left. + 2(H_{t++} - \cos \theta H_{0++})^2 \right. \\
&\quad \left. + 2(H_{t--} - \cos \theta H_{0--})^2 \right. \\
&\quad \left. + 2(H_{t00} - \cos \theta H_{000})^2 \right].
\end{aligned} \tag{20}$$

Integrating over $\cos \theta$ and summing over the lepton helicity, we can obtain the differential decay rate written as

$$\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{G_F^2 |V_{cb}|^2 |\vec{p}|}{96\pi^3 m_{B^*}^2} \frac{1}{3} q^2 \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \\
&\quad \times \left[\frac{3m_\ell^2}{2q^2} (H_{t++}^2 + H_{t--}^2 + H_{t00}^2) + (H_{+0+}^2 + H_{+0-}^2 \right. \\
&\quad \left. + H_{-0-}^2 + H_{-0+}^2 + H_{000}^2 + H_{0--}^2 + H_{0++}^2) \left(1 + \frac{m_\ell^2}{2q^2}\right) \right],
\end{aligned} \tag{21}$$

where the three nondiagonal interference terms in Eq. (20) vanish. In addition, paying attention to the polarization states of V meson, one can obtain the longitudinal differential decay width $d\Gamma^L/dq^2$ by picking out H_{t00}^2 , H_{+0-}^2 , H_{-0+}^2 and H_{000}^2 terms in Eq. (21).

Using Eqs. (19) and (20) given above, we can also construct some useful observables as follows. The q^2 -dependent ratios is defined as

$$R_V^{*(L)}(q^2) \equiv \frac{d\Gamma^L(\bar{B}^* \rightarrow V \tau^- \bar{\nu}_\tau)/dq^2}{d\Gamma^L(\bar{B}^* \rightarrow V \ell^- \bar{\nu}_\ell)/dq^2}, \tag{22}$$

where ℓ' denotes the light leptons μ and e (in the following calculations, we take $m_{e,\mu} = 0$). The lepton spin asymmetry and forward-backward asymmetry are defined as

$$A_{\lambda}^{*V}(q^2) = \frac{d\Gamma[\lambda_{\ell} = -1/2]/dq^2 - d\Gamma[\lambda_{\ell} = 1/2]/dq^2}{d\Gamma[\lambda_{\ell} = -1/2]/dq^2 + d\Gamma[\lambda_{\ell} = 1/2]/dq^2}, \quad (23)$$

and

$$A_{\theta}^{*V}(q^2) = \frac{\int_{-1}^0 d \cos \theta (d^2\Gamma/dq^2 d \cos \theta) - \int_0^1 d \cos \theta (d^2\Gamma/dq^2 d \cos \theta)}{d\Gamma/dq^2}, \quad (24)$$

respectively. These observables are independent of the CKM matrix elements, and the hadronic uncertainties canceled to a large extent, therefore, they can be predicted with a rather high accuracy.

3. Numerical Results and Discussions

In our numerical calculation, for the well-known Fermi coupling constant G_F and the masses of mesons and τ , we take their central values given by PDG [55]. For the CKM element, we take $|V_{cb}| = 41.80_{-0.60}^{+0.28} \times 10^{-3}$ given by CKMFitter Group [85]. In order to evaluate the branching fractions, the total decay widths (or lifetimes), $\Gamma_{\text{tot}}(B_{u,d,s,c}^*)$, are also essential inputs. However, there is no available experimental or theoretical information until now. While, due to the fact that the electromagnetic processes $B^* \rightarrow B\gamma$ dominates B^* decays, we can take the approximation $\Gamma_{\text{tot}}(B^*) \approx \Gamma(B^* \rightarrow B\gamma)$. In the light-front quark model (LFQM), the decay width of $B^* \rightarrow B\gamma$ decay is given by [86]

$$\begin{aligned} \Gamma(B^* \rightarrow B\gamma) &= \frac{\alpha}{3} [e_1 I(m_1, m_2, 0) + e_2 I(m_2, m_1, 0)]^2 \kappa_\gamma^3, \\ I(m_1, m_2, q^2) &= \int_0^1 \frac{dx}{8\pi^3} \int d^2k_{\perp} \frac{\psi(x, k'_{\perp}) \psi(x, k_{\perp})}{x \tilde{M}_0 \tilde{M}'_0} \\ &\times \left\{ \mathcal{A} + \frac{2}{\mathcal{M}_0} \left[k_{\perp}^2 - \frac{(k_{\perp} \cdot q_{\perp})^2}{q_{\perp}^2} \right] \right\}, \end{aligned} \quad (25)$$

where $\mathcal{A} = \bar{x}m_1 + xm_2$ with $\bar{x} = 1 - x$, $\mathcal{M}_0 = M_0 + m_1 + m_2$ with M_0 being the invariant mass of bound-state, α is the fine-structure constant, $\kappa_\gamma = (m_{B^*}^2 - m_B^2)/2m_{B^*}$ is the kinematically allowed energy of the outgoing photon. The radial wavefunction (WF) $\psi(x, k_{\perp})$ of bound-state is responsible for describing the momentum distribution of the constituent quarks. In this paper, we shall use the Gaussian-type WF

$$\psi(x, k_{\perp}) = 4 \frac{\pi^{3/4}}{\beta^{3/2}} \sqrt{\frac{\partial k_z}{\partial x}} \exp \left[-\frac{k_z^2 + k_{\perp}^2}{2\beta^2} \right], \quad (26)$$

where k_z is the relative momentum in z -direction and has the form $k_z = (x - 1/2)M_0 + m_2^2 - m_1^2/2M_0$. One can refer to Ref.

[86] for more details. Using the constituent quark masses and the Gaussian parameter β given in Table 1, we obtain the numerical results for $\Gamma(B^* \rightarrow B\gamma)$ as follows,

$$\Gamma_{\text{tot}}(B^{*+}) \approx \Gamma(B^{*+} \rightarrow B^+ \gamma) = (349 \pm 18) \text{eV}, \quad (27)$$

$$\Gamma_{\text{tot}}(B^{*0}) \approx \Gamma(B^{*0} \rightarrow B^0 \gamma) = (116 \pm 6) \text{eV}, \quad (28)$$

$$\Gamma_{\text{tot}}(B_s^{*0}) \approx \Gamma(B_s^{*0} \rightarrow B_s^0 \gamma) = (84_{-9}^{+11}) \text{eV}, \quad (29)$$

$$\Gamma_{\text{tot}}(B_c^{*+}) \approx \Gamma(B_c^{*+} \rightarrow B_c^0 \gamma) = (49_{-21}^{+28}) \text{eV}. \quad (30)$$

These theoretical predictions are generally in agreement with the ones obtained in the previous work based on different theoretical models [86–92].

Besides the inputs given above, the $B^* \rightarrow V$ transition form factors are also crucial inputs for evaluating observables, especially for the branching fraction. In this work, we adopt the covariant light-front quark model (CLFQM) [93–95] to evaluate their values. The theoretical formulas for the form factors of $V' \rightarrow V''$ have been given in our previous work (see Eqs. (39–48) in the appendix of Ref. [96]). These theoretical results are obtained within Drell-Yan-West frame, $q^+ = 0$, which implies that the form factors are known only for space-like momentum transfer, $q^2 = -q_{\perp}^2$, and the ones in the physical time-like region need an additional q^2 extrapolation. Following the strategy employed in Refs. [82, 93–95], one can parameterize the form factors as functions of q^2 by using dipole model in the space-like region and then extend them to the whole physical region $0 \leq q^2 \leq (m_{B^*} - m_V)^2$. The form factors in the dipole model have the form

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_{B^*}^2) + b(q^2/m_{B^*}^2)^2}, \quad (31)$$

where F denotes A_{1-4} and V_{1-6} . Using the inputs given in Table 1, we then present our theoretical prediction for the form factors of $\bar{B}^* \rightarrow D^*$, $\bar{B}_s^* \rightarrow D_s^*$, and $\bar{B}_c^* \rightarrow J/\psi$ transitions in Table 2. Their q^2 dependences are shown in Figure 1.

Using the formulas given in the last section and inputs given above, we then present our numerical results for the q^2 -integrated observables of $\bar{B}^* \rightarrow V\ell^-\bar{\nu}_{\ell}$ decays in Tables 3 and 4. For the branching fractions, the three errors in Table 3 are caused by the uncertainties of form factors, V_{cb} and $\Gamma_{\text{tot}}(B^*)$, respectively. For the other observables listed in Table 4, the theoretical uncertainties are caused only by the form factors. Besides, the q^2 dependence of differential decay rates $d\Gamma^{(L)}/dq^2$ and $A_{\lambda,\theta}^{*V}$, $R_V^{*(L)}$ are shown in Figures 2 and 3. The following are some analyses and discussions:

- (1) From Table 3, one can find a clear relation ($\bar{B}^{*-} \rightarrow D^{*0}\ell^-\bar{\nu}_{\ell}$): $B(\bar{B}^{*0} \rightarrow D^{*+}\ell^-\bar{\nu}_{\ell})$: $B(\bar{B}_s^{*0} \rightarrow D_s^{*+}\ell^-\bar{\nu}_{\ell})$: $B(\bar{B}_c^{*-} \rightarrow J/\psi\ell^-\bar{\nu}_{\ell}) \approx 1 : 3 : 4 : 6$, which is caused mainly by their total decay widths $\Gamma_{\text{tot}}(B^*)$ illustrated by Eqs. (27), (28), (29), and (30).

TABLE 1: The values of constituent quark masses and Gaussian parameters (in units of MeV) obtained by fitting to the data of decay constants [101, 102], where $q = u, d$.

$m_q = 250,$	$m_s = 450,$	$m_c = 1400,$	$m_b = 4640;$
$\beta_{b\bar{q}} = 540.7 \pm 9.6,$	$\beta_{b\bar{s}} = 601.9 \pm 7.4,$	$\beta_{b\bar{c}} = 933.9 \pm 11.1,$	for P -meson
$\beta_{c\bar{q}} = 413.0 \pm 12.0,$	$\beta_{c\bar{s}} = 514.1 \pm 18.5,$	$\beta_{c\bar{c}} = 684.4 \pm 6.7,$	
$\beta_{b\bar{q}} = 504.4 \pm 14.2,$	$\beta_{b\bar{s}} = 556.4 \pm 10.1,$	$\beta_{b\bar{c}} = 863.4 \pm 32.8,$	for V -meson

TABLE 2: The numerical results of form factors for $\bar{B}^* \rightarrow D^*$, $\bar{B}_s^* \rightarrow D_s^*$, and $\bar{B}_c^* \rightarrow J/\psi$ transitions within the CLFQM. The uncertainties are caused by the Gaussian parameters listed in Table 1.

$\bar{B}^* \rightarrow D^*$										
	A_1	A_2	A_3	A_4	V_1	V_2	V_3	V_4	V_5	V_6
$F(0)$	$0.66^{+0.01}_{-0.01}$	$0.36^{+0.00}_{-0.00}$	$0.07^{+0.00}_{-0.00}$	$0.08^{+0.00}_{-0.00}$	$0.67^{+0.01}_{-0.01}$	$0.36^{+0.00}_{-0.00}$	$0.13^{+0.00}_{-0.00}$	$0.00^{+0.00}_{-0.00}$	$1.17^{+0.01}_{-0.01}$	$0.48^{+0.01}_{-0.01}$
a	$1.31^{+0.02}_{-0.02}$	$1.32^{+0.02}_{-0.02}$	$1.79^{+0.02}_{-0.02}$	$1.81^{+0.02}_{-0.02}$	$1.30^{+0.02}_{-0.02}$	$1.32^{+0.02}_{-0.02}$	$1.72^{+0.02}_{-0.02}$	$-0.09^{+0.45}_{-0.40}$	$1.30^{+0.02}_{-0.02}$	$1.29^{+0.02}_{-0.02}$
b	$0.42^{+0.02}_{-0.02}$	$0.42^{+0.02}_{-0.02}$	$1.10^{+0.03}_{-0.03}$	$1.15^{+0.04}_{-0.04}$	$0.43^{+0.02}_{-0.02}$	$0.42^{+0.02}_{-0.02}$	$1.01^{+0.03}_{-0.04}$	$1.27^{+0.38}_{-0.28}$	$0.41^{+0.02}_{-0.02}$	$0.40^{+0.02}_{-0.02}$
$\bar{B}_s^* \rightarrow D_s^*$										
$F(0)$	$0.65^{+0.01}_{-0.01}$	$0.38^{+0.01}_{-0.01}$	$0.10^{+0.00}_{-0.00}$	$0.09^{+0.00}_{-0.00}$	$0.66^{+0.01}_{-0.01}$	$0.38^{+0.01}_{-0.01}$	$0.15^{+0.00}_{-0.00}$	$-0.02^{+0.00}_{-0.00}$	$1.19^{+0.02}_{-0.02}$	$0.53^{+0.01}_{-0.01}$
a	$1.42^{+0.03}_{-0.04}$	$1.47^{+0.03}_{-0.03}$	$1.89^{+0.03}_{-0.03}$	$1.88^{+0.02}_{-0.03}$	$1.43^{+0.03}_{-0.04}$	$1.48^{+0.03}_{-0.03}$	$1.79^{+0.03}_{-0.03}$	$2.22^{+0.04}_{-0.03}$	$1.41^{+0.03}_{-0.03}$	$1.35^{+0.04}_{-0.04}$
b	$0.64^{+0.04}_{-0.05}$	$0.67^{+0.04}_{-0.04}$	$1.33^{+0.05}_{-0.06}$	$1.36^{+0.09}_{-0.07}$	$0.64^{+0.04}_{-0.05}$	$0.67^{+0.04}_{-0.05}$	$1.20^{+0.06}_{-0.06}$	$1.92^{+0.08}_{-0.12}$	$0.61^{+0.04}_{-0.05}$	$0.56^{+0.04}_{-0.05}$
$\bar{B}_c^* \rightarrow J/\psi$										
$F(0)$	$0.55^{+0.01}_{-0.01}$	$0.35^{+0.00}_{-0.00}$	$0.14^{+0.00}_{-0.00}$	$0.15^{+0.01}_{-0.01}$	$0.57^{+0.01}_{-0.01}$	$0.35^{+0.00}_{-0.00}$	$0.21^{+0.00}_{-0.01}$	$-0.01^{+0.01}_{-0.01}$	$1.19^{+0.02}_{-0.02}$	$0.64^{+0.01}_{-0.01}$
a	$2.48^{+0.07}_{-0.07}$	$2.65^{+0.08}_{-0.08}$	$2.88^{+0.09}_{-0.09}$	$2.88^{+0.08}_{-0.08}$	$2.48^{+0.07}_{-0.07}$	$2.56^{+0.08}_{-0.08}$	$2.75^{+0.08}_{-0.09}$	$3.58^{+0.17}_{-0.12}$	$2.42^{+0.07}_{-0.07}$	$2.32^{+0.06}_{-0.06}$
b	$2.71^{+0.20}_{-0.22}$	$2.87^{+0.23}_{-0.26}$	$3.88^{+0.31}_{-0.34}$	$3.90^{+0.30}_{-0.33}$	$2.73^{+0.20}_{-0.22}$	$2.88^{+0.23}_{-0.26}$	$3.51^{+0.29}_{-0.32}$	$6.37^{+0.23}_{-0.13}$	$2.54^{+0.20}_{-0.22}$	$2.33^{+0.17}_{-0.19}$

In Table 3, the previous predictions based on the Bethe-Salpeter (BS) method [67] and the assumption of heavy quark symmetry (HQS) [68] are also listed for comparison. It can be found that the results based on the BS method and the assumption of HQS are a little bit smaller and larger respectively than our results, but they are also in agreement in the order of magnitude. These $b \rightarrow c \ell^- \bar{\nu}_\ell$ induced B^* weak decays have the branching fractions of the order $\mathcal{O}(10^{-8} - 10^{-7}) > 10^{-9}$, and therefore are in the scope of Belle-II or LHC experiments. In addition, due to the fact that $V_{ub}/V_{cb} \approx 0.088$, the $b \rightarrow u \ell^- \bar{\nu}_\ell$ induced B^* weak decays should have much smaller branching fractions, which are at the level of $\mathcal{O}(10^{-10} - 10^{-9})$, and thus are hard to be observed in the near future.

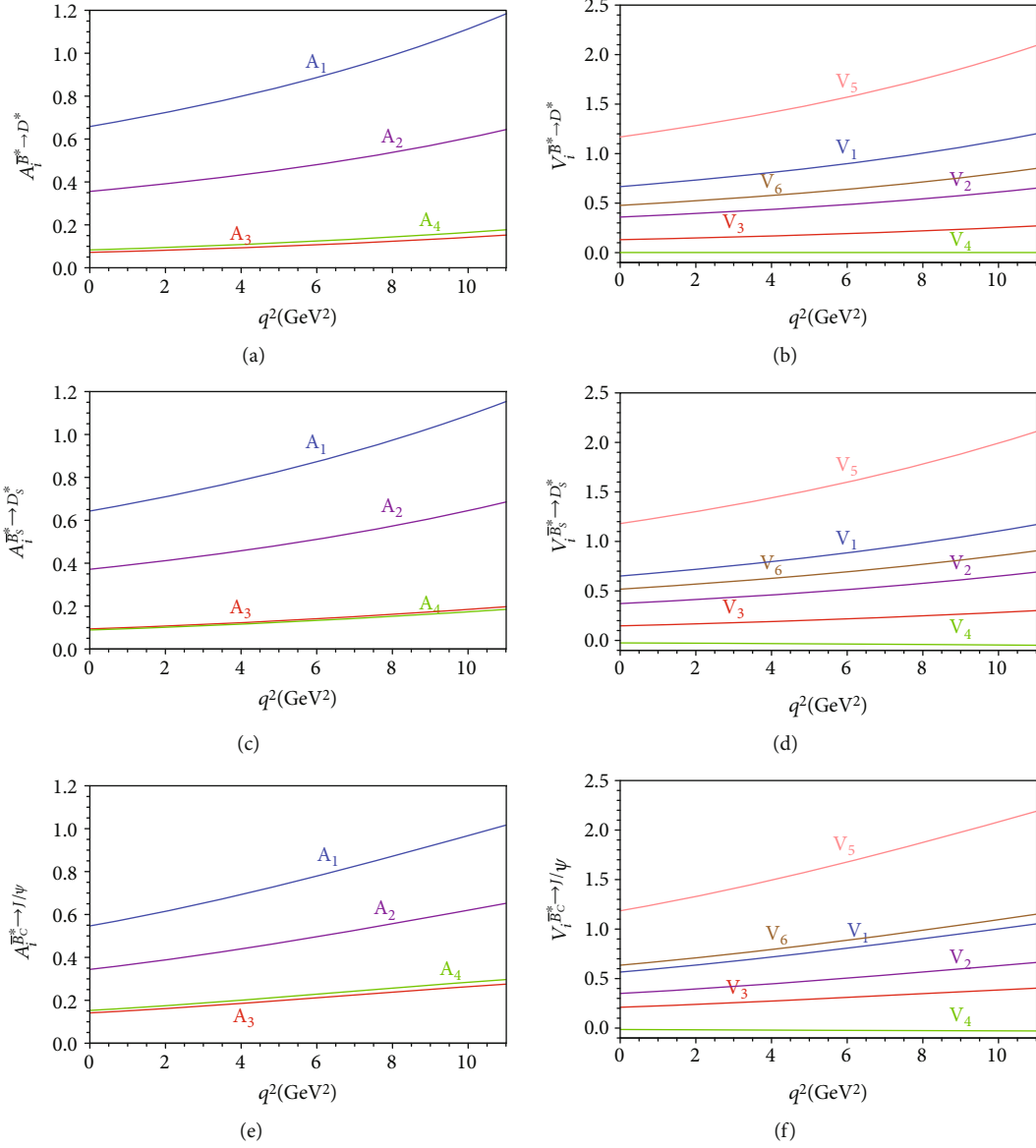
- (2) Deviations from the SM predictions in $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ decay modes have been observed by the BaBar [11, 12], Belle [13–15], and LHCb [16, 17] collaborations in the ratios $R_{D^{(*)}} \equiv \mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau) / \mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell'^- \bar{\nu}_{\ell'})$ ($\ell' = \mu, e$). The combination of these measurements performed by the Heavy Flavour Averaging Group (HFLAV) [1] reads

$$R_D^{\text{HFLAV}} = 0.407 \pm 0.039 \pm 0.024, R_{D^*}^{\text{HFLAV}} = 0.306 \pm 0.013 \pm 0.007, \quad (32)$$

which show tensions of about 2 and 4 σ , respectively, with the SM predictions [97]. Very recent measurement of R_{D^*} by Belle [98] results in values more compatible with the SM and yields a downward shift in the average. However, even though such measurement is included in the global average, the deviation is still larger than 3 σ [97]. If this “ R_{D^*} anomaly” is the truth, it possibly exists also in the $b \rightarrow c$ induced $\bar{B}^* \rightarrow V \ell \bar{\nu}_\ell$ decays, which therefore can provide another useful test on the lepton flavor universality and the various method based on the SM and NP for resolving “ R_{D^*} anomaly.” Our numerical results for $R_V^{*(L)}$ are summarized in Table 4, and the q^2 -spectra of $R_V^{*(L)}$ are shown in Figure 3. It can be found that

$$R_{D^*}^{*(L)} \simeq R_{D_s^*}^{*(L)} \simeq R_{\psi}^{*(L)}, \quad (33)$$

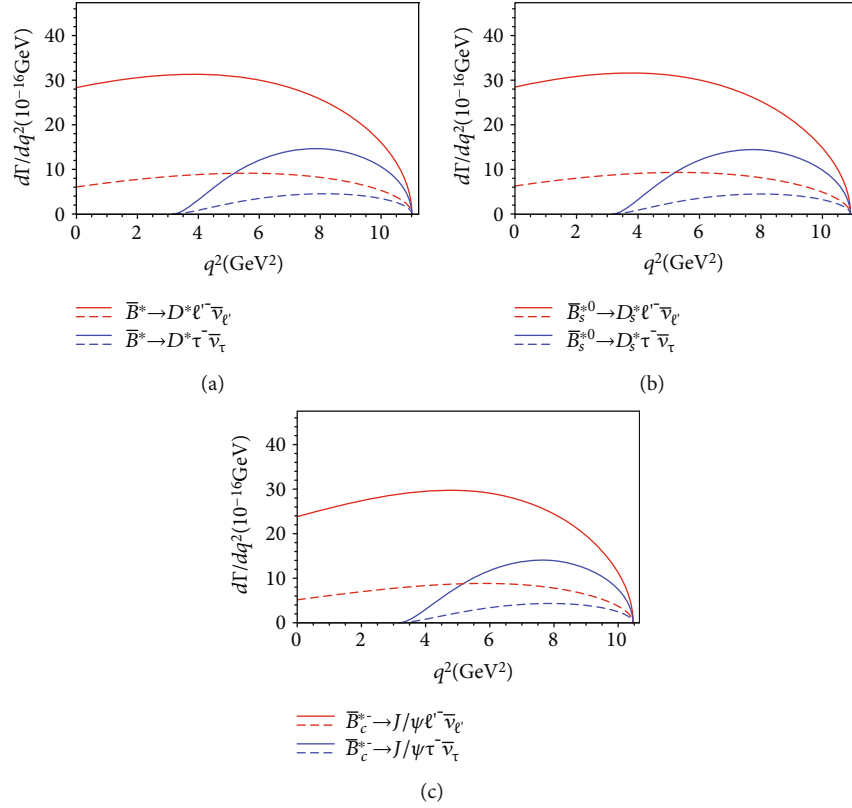
within theoretical uncertainties. Moreover, their q^2 -spectra almost overlap with each other as shown in Figures 3(a)

FIGURE 1: The q^2 -dependences of form factors for $\bar{B}^* \rightarrow D^*$, $\bar{B}_s^* \rightarrow D_s^*$, and $\bar{B}_c^* \rightarrow J/\psi$ transitions.TABLE 3: The SM predictions for the branching fractions of $\bar{B}^* \rightarrow V\ell^-\bar{\nu}_\ell$ decays.

Decay mode	This work	BS method [67]	HQS [68]
$\bar{B}^{*-} \rightarrow D^{*0}\ell'^-\bar{\nu}_{\ell'}$	$8.42_{-0.24-0.24-0.42}^{+0.23+0.11+0.45} \times 10^{-8}$	1.26×10^{-7}	6.41×10^{-8}
$\bar{B}^{*-} \rightarrow D^{*0}\tau^-\bar{\nu}_\tau$	$2.26_{-0.08-0.06-0.11}^{+0.08+0.03+0.12} \times 10^{-8}$	2.74×10^{-8}	1.29×10^{-8}
$\bar{B}^{*0} \rightarrow D^{*+}\ell'^-\bar{\nu}_{\ell'}$	$2.51_{-0.07-0.07-0.12}^{+0.08+0.03+0.13} \times 10^{-7}$	–	1.92×10^{-7}
$\bar{B}^{*0} \rightarrow D^{*+}\tau^-\bar{\nu}_\tau$	$6.73_{-0.25-0.19-0.32}^{+0.24+0.09+0.35} \times 10^{-8}$	–	3.88×10^{-8}
$\bar{B}_s^{*0} \rightarrow D_s^{*+}\ell'^-\bar{\nu}_{\ell'}$	$3.46_{-0.17-0.10-0.41}^{+0.17+0.05+0.41} \times 10^{-7}$	4.63×10^{-7}	2.53×10^{-7}
$\bar{B}_s^{*0} \rightarrow D_s^{*+}\tau^-\bar{\nu}_\tau$	$9.10_{-0.59-0.26-1.07}^{+0.60+0.12+1.07} \times 10^{-8}$	1.05×10^{-7}	5.05×10^{-8}
$\bar{B}_c^{*-} \rightarrow J/\psi\ell'^-\bar{\nu}_{\ell'}$	$5.44_{-0.33-0.16-2.00}^{+0.33+0.07+4.06} \times 10^{-7}$	5.37×10^{-7}	2.91×10^{-7}
$\bar{B}_c^{*-} \rightarrow J/\psi\tau^-\bar{\nu}_\tau$	$1.43_{-0.12-0.04-0.52}^{+0.13+0.02+1.07} \times 10^{-7}$	1.49×10^{-7}	5.65×10^{-8}

TABLE 4: Predictions for q^2 -integrated observables $A_{\lambda,\theta}^{*V}(\ell = \tau)$, $R_V^{*(L)}$, and F_L^{*V} .

Obs.	Prediction	Obs.	Prediction	Obs.	Prediction
$A_\lambda^{*D^*}$	$0.237^{+0.017}_{-0.016}$	$A_\lambda^{*D_s^*}$	$0.231^{+0.029}_{-0.030}$	$A_\lambda^{*J/\psi}$	$0.214^{+0.043}_{-0.040}$
$A_\theta^{*D^*}$	$0.070^{+0.007}_{-0.006}$	$A_\theta^{*D_s^*}$	$0.071^{+0.011}_{-0.011}$	$A_\theta^{*J/\psi}$	$0.078^{+0.014}_{-0.013}$
$R_{D^*}^*$	$0.269^{+0.003}_{-0.003}$	$R_{D_s^*}^*$	$0.263^{+0.005}_{-0.005}$	$R_{J/\psi}^*$	$0.262^{+0.009}_{-0.007}$
$R_{D^*}^{*L}$	$0.285^{+0.004}_{-0.003}$	$R_{D_s^*}^{*L}$	$0.277^{+0.006}_{-0.007}$	$R_{J/\psi}^{*L}$	$0.278^{+0.009}_{-0.009}$
$F_L^{*D^*}$	$0.304^{+0.003}_{-0.004}$	$F_L^{*D_s^*}$	$0.306^{+0.005}_{-0.006}$	$F_L^{*J/\psi}$	$0.303^{+0.006}_{-0.007}$

FIGURE 2: The q^2 -dependences of differential decay rates $d\Gamma/dq^2$ (solid lines) and $d\Gamma^L/dq^2$ (dashed lines).

and 3(b). Using the results summarized in Table 3, we can also obtain the predictions based on the BS method and HQS,

$$\begin{aligned}
 R_{D^*}^* &= 0.217, \\
 R_{D_s^*}^* &= 0.227, \\
 R_{J/\psi}^* &= 0.277, \text{BSmethod} \\
 R_{D^*}^{*L} &= 0.202, \\
 R_{D_s^*}^{*L} &= 0.200, \\
 R_{J/\psi}^{*L} &= 0.194. \text{HQS}
 \end{aligned} \tag{34}$$

It can be found that these results are different from our predictions more or less because different models and parameterizations are used for evaluating form factors, which

has been observed in the case of R_{D^*} [18]. Future measurement will make a judgement on these results.

- (3) Besides, the lepton spin asymmetry and the forward-backward asymmetry are also important observables for testing the SM and NP scenarios, for instance, two-Higgs-doublet models, and R-parity violating supersymmetry models [42–47], because their theoretical uncertainties can be well controlled and the zero-crossing points of their q^2 -spectra are sensitive to the NP effects [42]. Our numerical results for q^2 -integrated $A_{\lambda,\theta}^{*V}$ and A_θ^{*V} are collected in Table 4, and the q^2 dependences of $A_{\lambda,\theta}^{*V}(q^2)$ and A_θ^{*V} are shown by Figures 3(c) and 3(d). One can easily find that $A_{\lambda,\theta}^{*D^*} \approx A_{\lambda,\theta}^{*D_s^*} \approx A_{\lambda,\theta}^{*J/\psi}$. Moreover, the q^2 -spectra of $A_{\lambda,\theta}^{*V}$ are almost overlapping with each other as

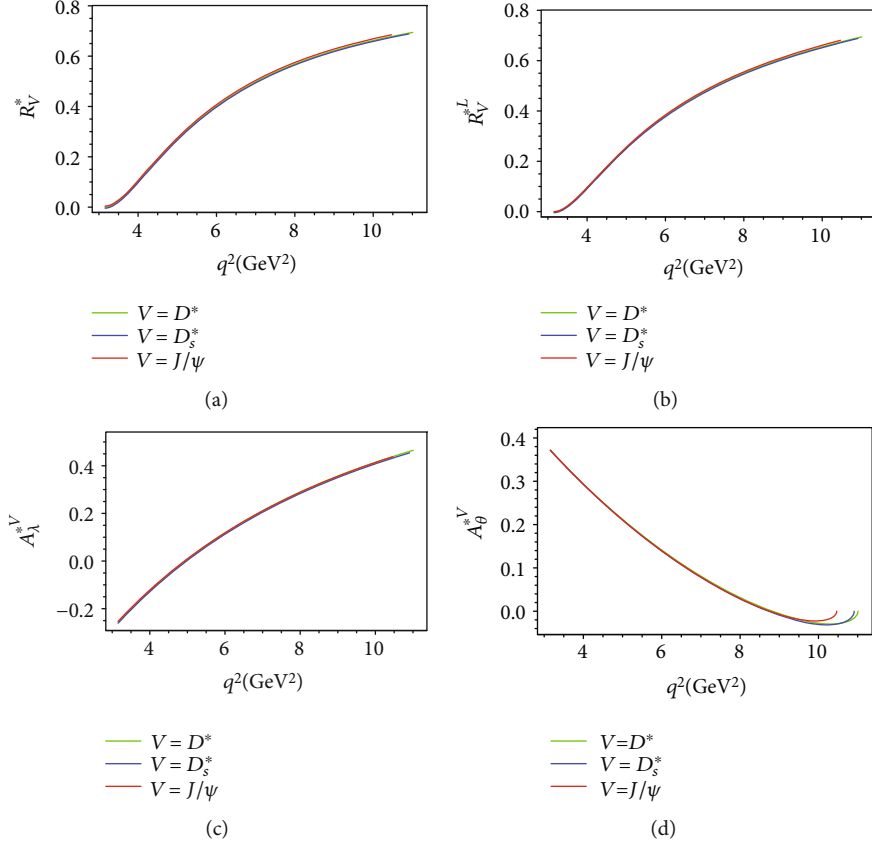


FIGURE 3: The q^2 -dependences of $R_V^{*(L)}(q^2)$, $A_\theta^{*V}(q^2)$, and $A_\lambda^{*V}(q^2)$.

shown in Figure 3(c), and the case of A_θ^{*V} is similar except that the q^2 -spectrum of $A_\theta^{*J/\psi}$ deviates from the ones of $A_\theta^{*D^*}$ and $A_\theta^{*D_s^*}$ at large q^2 . In addition, A_λ^{*V} crosses the zero point at $q^2 \approx 5 \text{ GeV}^2$, however, A_θ^{*V} does not have the zero point in all q^2 region.

- (4) The D^* longitudinal polarization fraction in semileptonic $B^0 \rightarrow D^{*+} \tau^+ \nu_\tau$ decay, defined as $F_L^{D^*} = \Gamma_{\lambda_{D^*}=0}(B^0 \rightarrow D^{*+} \tau^+ \nu_\tau) / \Gamma(B^0 \rightarrow D^{*+} \tau^+ \nu_\tau)$, has been measured by Belle experiment with $F_L^{D^*} = 0.60 \pm 0.08(\text{stat.}) \pm 0.04(\text{syst.})$ [99], which deviates from the SM prediction $(F_L^{D^*})_{SM} = 0.457 \pm 0.010$ [100] by 1.6σ . Similarly, we can define the longitudinal polarization fraction

$$F_L^{*V} = \frac{\Gamma_{\lambda_{D^*}=0}(\bar{B}^* \rightarrow V \tau^- \bar{\nu}_\tau)}{\Gamma(\bar{B}^* \rightarrow V \tau^- \bar{\nu}_\tau)} \quad (35)$$

for $\bar{B}^* \rightarrow V \tau^- \bar{\nu}_\tau$ decay modes. From the numerical results given in the last row of Table 4, one can easily find that

$$F_L^{*D^*} \simeq F_L^{*D_s^*} \simeq F_L^{*J/\psi} \simeq 30\%, \quad (36)$$

which implies that $\bar{B}^* \rightarrow V \tau^- \bar{\nu}_\tau$ decay is dominated by the transverse polarization. It is obviously different from the cor-

responding $\bar{B} \rightarrow V \tau^- \bar{\nu}_\tau$ decay mode, which is dominated by the longitudinal polarization state.

4. Summary

In this paper, motivated by abundant B^* data samples at high-luminosity heavy-flavor experiments in the future, we have studied the $b \rightarrow c$ induced $\bar{B}_{u,d,s,c}^* \rightarrow V \ell^- \bar{\nu}_\ell$ ($V = D_{u,d}^*, D_s^*, (J/\psi)$ and $\ell = e, \mu, \tau$) decays within the SM. The helicity amplitudes are investigated in detail, and the form factors of $\bar{B}^* \rightarrow V$ transitions are computed within the covariant light-front quark model. After that, we present our predictions for the observables including branching fraction (decay width), leptonic spin asymmetry, forward-backward asymmetry, ratio $R_V^{*(L)}$, and longitudinal polarization fraction in Tables 3 and 4 and Figures 2 and 3. It is found that all these semileptonic B^* decays have relatively large branching fractions of $O(10^{-8}) \sim O(10^{-7})$, in which $\mathcal{B}(\bar{B}_c^* \rightarrow J/\psi \ell'^- \bar{\nu}_{\ell'}) \sim 5 \times 10^{-7}$ is the largest one, and are hopeful to be observed at running LHC and SuperKEKB/Belle-II experiments; in addition, for the $\bar{B}^* \rightarrow V \tau^- \bar{\nu}_\tau$ decay, the longitudinal polarization state of V meson presents only about 30% contribution to the integrated decay width, which is obviously different from the corresponding $\bar{B} \rightarrow V \tau^- \bar{\nu}_\tau$ decay. All of results and findings in this paper are waiting for the experimental test in the future.

Data Availability

This manuscript has no associated data or the data will not be deposited. This is a theoretical research work, no additional data are associated with this work.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

- [1] Heavy Flavor Averaging Group (HFLAV), “Averages of b-hadron, c-hadron, and τ -lepton properties as of summer 2016,” *The European Physical Journal C*, vol. 77, no. 12, p. 895, 2017.
- [2] S. Descotes-Genon, J. Matias, M. Ramon, and J. Virto, “Implications from clean observables for the binned analysis of $B \rightarrow K^* \mu^+ \mu^-$ at large recoil,” *Journal of High Energy Physics*, vol. 2013, no. 1, p. 48, 2013.
- [3] The LHCb Collaboration, “Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using 3 fb⁻¹ of integrated luminosity,” *Journal of High Energy Physics*, vol. 2016, no. 2, p. 104, 2016.
- [4] CMS Collaboration, “Angular analysis of the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ from pp collisions at $\sqrt{s} = 8$ TeV,” *Physics Letters B*, vol. 753, pp. 424–448, 2016.
- [5] ATLAS Collaboration, “Angular analysis of $B_d^0 \rightarrow K^* \mu^+ \mu^-$ decays in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector,” *Journal of High Energy Physics*, vol. 1810, no. 10, p. 47, 2018.
- [6] Belle Collaboration, “Lepton-flavor-dependent angular analysis of $B \rightarrow K^* \ell^+ \ell^-$,” *Physical Review Letters*, vol. 118, no. 11, article 111801, 2017.
- [7] The LHCb Collaboration, “Angular analysis and differential branching fraction of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$,” *Journal of High Energy Physics*, vol. 2015, no. 9, p. 179, 2015.
- [8] The LHCb Collaboration, “Differential branching fraction and angular analysis of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$,” *Journal of High Energy Physics*, vol. 2013, no. 7, p. 84, 2013.
- [9] A. J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab, “ $B \rightarrow \pi\pi$, New Physics in $B \rightarrow \pi K$, and Implications for Rare K and B Decays,” *Physical Review Letters*, vol. 92, no. 10, article 101804, 2004.
- [10] A. J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab, “Anatomy of prominent B and K decays and signatures of CP-violating new physics in the electroweak penguin sector,” *Nuclear Physics B*, vol. 697, no. 1-2, pp. 133–206, 2004.
- [11] BABAR Collaboration, “Evidence for an excess of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ decays,” *Physical Review Letters*, vol. 109, no. 10, article 101802, 2012.
- [12] BaBar Collaboration, “Measurement of an excess of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ decays and implications for charged Higgs bosons,” *Physical Review D*, vol. 88, no. 7, article 072012, 2013.
- [13] Belle Collaboration, “Measurement of the branching ratio of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ relative to $\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell$ decays with hadronic tagging at Belle,” *Physical Review D*, vol. 92, no. 7, article 072014, 2015.
- [14] Belle Collaboration, “Measurement of the branching ratio of $\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau$ relative to $\bar{B}^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$ decays with a semi-leptonic tagging method,” *Phys. Rev. D*, vol. 94, no. 7, article 072007, 2016.
- [15] Belle Collaboration, “Measurement of the τ lepton polarization in the decay $\bar{B} \rightarrow D^* \tau^- \bar{\nu}_\tau$,” <http://arxiv.org/abs/1608.06391>.
- [16] LHCb Collaboration, “Measurement of the Ratio of Branching Fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau) / \mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu)$,” *Physical Review Letters*, vol. 115, no. 11, article 111803, 2015 Addendum: [Phys. Rev. Lett. 115 (2015) no. 15, 159901].
- [17] LHCb Collaboration, “Measurement of the Ratio of the $B^0 \rightarrow D^{*+} \tau^+ \nu_\tau$ and $B^0 \rightarrow D^{*+} \mu^+ \nu_\mu$ branching fractions using three-prong τ -lepton decays,” *Physical Review Letters*, vol. 120, no. 17, article 171802, 2018.
- [18] S. Jaiswal, S. Nandi, and S. K. Patra, “Extraction of $|V_{cb}|$ from $B \rightarrow D^{(*)} \ell \nu_\ell$ and the standard model predictions of $R(D^{(*)})$,” *Journal of High Energy Physics*, vol. 2017, no. 12, p. 60, 2017.
- [19] S. Jaiswal, S. Nandi, and S. K. Patra, “Updates on SM predictions of $|V_{cb}|$ and $R(D^*)$ in $B \rightarrow D^* \ell \nu_\ell$ decays,” <http://arxiv.org/abs/2002.05726>.
- [20] B. Bhattacharya, A. Datta, D. London, and S. Shivashankara, “Simultaneous explanation of the R_K and $R(D^{(*)})$ puzzles,” *Physics Letters B*, vol. 742, pp. 370–374, 2015.
- [21] S. Bhattacharya, S. Nandi, and S. K. Patra, “Optimal-observable analysis of possible new physics in $B \rightarrow D^{(*)} \tau \nu_\tau$,” *Physics Letters D*, vol. 93, no. 3, article 034011, 2016.
- [22] C. H. Chen and C. Q. Geng, “Lepton angular asymmetries in semileptonic charmful B decays,” *Physics Letters D*, vol. 71, no. 7, article 077501, 2005.
- [23] S. Bhattacharya, S. Nandi, and S. K. Patra, “Looking for possible new physics in $B \rightarrow D^{(*)} \tau \nu_\tau$ in light of recent data,” *Physical Review D*, vol. 95, no. 7, article 075012, 2017.
- [24] S. Bhattacharya, S. Nandi, and S. K. Patra, “ $b \rightarrow c \tau \nu_\tau$ decays: a catalogue to compare, constrain, and correlate new physics effects,” *The European Physical Journal C*, vol. 79, no. 3, p. 268, 2019.
- [25] A. K. Alok, D. Kumar, J. Kumar, S. Kumbhakar, and S. U. Sankar, “New physics solutions for R_D and R_{D^*} ,” *Journal of High Energy Physics*, vol. 2018, no. 9, p. 152, 2018.
- [26] F. Feruglio, P. Paradisi, and O. Sumensari, “Implications of scalar and tensor explanations of R_{D^*} ,” *Journal of High Energy Physics*, vol. 2018, no. 11, p. 191, 2018.
- [27] A. Azatov, D. Bardhan, D. Ghosh, F. Sgarlata, and E. Venturini, “Anatomy of $b \rightarrow c \tau \nu$ anomalies,” *Journal of High Energy Physics*, vol. 2018, no. 11, p. 187, 2018.
- [28] M. Freytsis, Z. Ligeti, and J. T. Ruderman, “Flavor models for $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$,” *Physical Review D*, vol. 92, no. 5, article 054018, 2015.
- [29] X. Q. Li, Y. D. Yang, and X. Zhang, “Revisiting the one leptoquark solution to the $R(D^{(*)})$ anomalies and its phenomenological implications,” *Journal of High Energy Physics*, vol. 2016, no. 8, p. 54, 2016.
- [30] G. Hiller, D. Loose, and K. Schönwald, “Leptoquark flavor patterns & B decay anomalies,” *Journal of High Energy Physics*, vol. 2016, no. 12, p. 27, 2016.

- [31] M. Blanke and A. Crivellin, “ B meson anomalies in a Pati-Salam model within the Randall-Sundrum background,” *Physical Review Letters*, vol. 121, no. 1, article 011801, 2018.
- [32] C. S. Kim, Y. W. Yoon, and X. B. Yuan, “Exploring top quark FCNC within 2HDM type III in association with flavor physics,” *Journal of High Energy Physics*, vol. 2015, no. 12, pp. 1–30, 2015.
- [33] A. Crivellin, J. Heeck, and P. Stoffer, “Perturbed lepton-specific two-higgs-doublet model facing experimental hints for physics beyond the standard model,” *Physical Review Letters*, vol. 116, no. 8, article 081801, 2016.
- [34] X. G. He and G. Valencia, “Lepton universality violation and right-handed currents in $b \rightarrow crv$,” *Physics Letters B*, vol. 779, pp. 52–57, 2018.
- [35] Q. Y. Hu, Y. D. Yang, and M. D. Zheng, “Revisiting the B -physics anomalies in R -parity violating MSSM,” <http://arxiv.org/abs/2002.09875>.
- [36] D. Y. Wang, Y. D. Yang, and X. B. Yuan, “ $b \rightarrow c\tau\bar{\nu}$ decay in supersymmetry with R -parity violation,” *Chinese Physics C*, vol. 43, no. 8, article 083103, 2019.
- [37] H. Yan, Y. D. Yang, and X. B. Yuan, “Phenomenology of $b \rightarrow c\tau\bar{\nu}$ decays in a scalar leptoquark model,” *Chinese Physics C*, vol. 43, no. 8, article 083105, 2019.
- [38] Q. Y. Hu, X. Q. Li, and Y. D. Yang, “ $b \rightarrow c\tau\nu$ transitions in the standard model effective field theory,” *The European Physical Journal C*, vol. 79, no. 3, p. 264, 2019.
- [39] S. P. Li, X. Q. Li, Y. D. Yang, and X. Zhang, “ $R_{D^{(*)}}, R_{K^{(*)}}$ and neutrino mass in the 2HDM-III with right-handed neutrinos,” *Journal of High Energy Physics*, vol. 2018, no. 9, p. 149, 2018.
- [40] W. Altmannshofer, P. S. Bhupal Dev, and A. Soni, “ $R_{D^{(*)}}$ anomaly: a possible hint for natural supersymmetry with R -parity violation,” *Physical Review D*, vol. 96, no. 9, article 095010, 2017.
- [41] K. Cheung, Z. R. Huang, H. D. Li, C. D. Lü, Y. N. Mao, and R. Y. Tang, “Revisit to the $b \rightarrow c\tau\nu$ transition: in and beyond the SM,” <http://arxiv.org/abs/2002.07272>.
- [42] A. Celis, M. Jung, X. Q. Li, and A. Pich, “Sensitivity to charged scalars in $B \rightarrow D^{(*)}\tau\nu_\tau$ and $B \rightarrow \tau\nu_\tau$ decays,” *Journal of High Energy Physics*, vol. 2013, no. 1, article 5379, 2013.
- [43] A. Celis, M. Jung, X. Q. Li, and A. Pich, “Scalar contributions to $b \rightarrow c(u)\tau\nu$ transitions,” *Physics Letters B*, vol. 771, pp. 168–179, 2017.
- [44] X. Q. Li, “ $R(D)$ and $R(D^*)$ anomalies and their phenomenological implications,” *Nuclear and Particle Physics Proceedings*, vol. 287–288, pp. 181–184, 2017.
- [45] R. M. Wang, J. Zhu, H. M. Gan, Y. Y. Fan, Q. Chang, and Y. G. Xu, “Probing R -parity violating supersymmetric effects in the exclusive $b \rightarrow c\ell^-\bar{\nu}_\ell$ decays,” *Physical Review D*, vol. 93, no. 9, article 094023, 2016.
- [46] J. Zhu, B. Wei, J. H. Sheng, R. M. Wang, Y. Gao, and G. R. Lu, “Probing the R -parity violating supersymmetric effects in $B_c \rightarrow J/\psi\ell - \bar{\nu}_\ell, \eta\ell - \bar{\nu}_\ell$ and $\Lambda_b \rightarrow \Lambda_c\ell - \bar{\nu}_\ell$ decays,” *Nuclear Physics B*, vol. 934, pp. 380–395, 2018.
- [47] Q. Y. Hu, X. Q. Li, Y. Muramatsu, and Y. D. Yang, “ R -parity violating solutions to the $RD^{(*)}$ anomaly and their GUT-scale,” *Physical Review D*, vol. 99, no. 1, article 015008, 2019.
- [48] Z. R. Huang, Y. Li, C. D. Lu, M. A. Paracha, and C. Wang, “Footprints of new physics in $b \rightarrow c\tau\nu$ transitions,” *Physical Review D*, vol. 98, no. 9, article 095018, 2018.
- [49] S. Bifani, S. Descotes-Genon, A. R. Vidal, and M. H. Schune, “Review of lepton universality tests in B decays,” *Journal of Physics G: Nuclear and Particle Physics*, vol. 46, no. 2, article 023001, 2019.
- [50] Y. Li and C. D. Lü, “Recent anomalies in B physics,” *Science Bulletin*, vol. 63, no. 5, pp. 267–269, 2018.
- [51] N. Isgur and M. B. Wise, “Spectroscopy with heavy-quark symmetry,” *Physical Review Letters*, vol. 66, no. 9, pp. 1130–1133, 1991.
- [52] S. Godfrey and R. Kokoski, “Properties of P -wave mesons with one heavy quark,” *Physical Review D*, vol. 43, no. 5, pp. 1679–1687, 1991.
- [53] E. J. Eichten, C. T. Hill, and C. Quigg, “Properties of orbitally excited heavy-light (Qq^*) mesons,” *Physical Review Letters*, vol. 71, no. 25, pp. 4116–4119, 1993.
- [54] D. Ebert, V. O. Galkin, and R. N. Faustov, “Mass spectrum of orbitally and radially excited heavy-light mesons in the relativistic quark model,” *Physical Review D*, vol. 57, no. 9, pp. 5663–5669, 1998, [Erratum *Phys. Rev. D* 59 (1998) 019902].
- [55] M. Tanabashi, K. Hagiwara, K. Hikasa et al., “Review of Particle Physics,” *Physical Review D*, vol. 98, no. 3, article 030001, 2018.
- [56] Belle-II Collaboration, “Belle II Technical Design Report,” <http://arxiv.org/abs/1011.0352>.
- [57] CLEO Collaboration [hep-ex/0607080](http://arxiv.org/abs/hep-ex/0607080).
- [58] R. Aaij, C. Abellan Beteta, B. Adeva et al., “Measurement of $\sigma(pp \rightarrow b\bar{b}X)$ at $\sqrt{s} = 7\text{TeV}$ in the forward region,” *Physics Letters B*, vol. 694, no. 3, pp. 209–216, 2010.
- [59] A. Bharucha, The LHCb Collaboration, I. I. Bigi et al., “Implications of LHCb measurements and future prospects,” *The European Physical Journal C*, vol. 73, no. 4, p. 2373, 2013.
- [60] The LHCb Collaboration, “LHCb detector performance,” *International Journal of Modern Physics A*, vol. 30, no. 7, p. 1530022, 2015.
- [61] B. Grinstein and J. Martin Camalich, “Weak decays of excited B mesons,” *Physical Review Letters*, vol. 116, no. 14, article 141801, 2016.
- [62] A. Khodjamirian, T. Mannel, and A. A. Petrov, “Direct probes of flavor-changing neutral currents in e^+e^- collisions,” *Journal of High Energy Physics*, vol. 2015, no. 11, article 2556, 2015.
- [63] G. Z. Xu, Y. Qiu, C. P. Shen, and Y. J. Zhang, “ $B_{s,d}^* \rightarrow \mu^+\mu^-$ and its impact on $B_{s,d} \rightarrow \mu^+\mu^-$,” *The European Physical Journal C*, vol. 76, no. 11, 2016.
- [64] Z. G. Wang, “Semileptonic Decays $B_c^* \rightarrow \eta_c\ell\bar{\nu}_\ell$ with QCD Sum Rules,” *Communications in Theoretical Physics*, vol. 61, no. 1, pp. 81–88, 2014.
- [65] K. Zeynali, V. Bashiry, and F. Zolfagharpour, “Form factors and decay rate of $B_c^* \rightarrow D_s^*\ell^+\ell^-$ decays in the QCD sum rules,” *The European Physical Journal A*, vol. 50, no. 8, 2014.
- [66] V. Bashiry, “Investigation of the Rare Exclusive Decays in the Framework of the QCD Sum Rules,” *Advances in High Energy Physics*, vol. 2014, Article ID 503049, 10 pages, 2014.
- [67] T. Wang, Y. Jiang, T. Zhou, X. Z. Tan, and G. L. Wang, “Semileptonic decays of B^*, B_s^* , and B_c^* with the Bethe-Salpeter method,” *Journal of Physics G: Nuclear and Particle Physics*, vol. 45, no. 11, article 115001, 2018.
- [68] L. R. Dai, X. Zhang, and E. Oset, “Semileptonic decays of $B^{(*)}, D^{(*)}$ into $\nu\ell$ and pseudoscalar or vector mesons,” *Physical Review D*, vol. 98, no. 3, article 036004, 2018.

- [69] Q. Chang, J. Zhu, X. L. Wang, J. F. Sun, and Y. L. Yang, "Study of semileptonic $\bar{B}^* \rightarrow P\ell\bar{\nu}_\ell$ decays," *Nuclear Physics B*, vol. 909, pp. 921–933, 2016.
- [70] Q. Chang, P. P. Li, X. H. Hu, and L. Han, "Study of nonleptonic $B_{(s)}^* \rightarrow M_1 M_2$ ($M = D, D_s, \pi, K$) weak decays with factorization approach," *International Journal of Modern Physics A*, vol. 30, no. 27, article 1550162, 2015.
- [71] Q. Chang, X. Hu, J. Sun, X. Wang, and Y. Yang, "Study of Nonleptonic and Weak Decays," *Advances in High Energy Physics*, vol. 2015, Article ID 767523, 8 pages, 2015.
- [72] Q. Chang, L. X. Chen, Y. Y. Zhang, J. F. Sun, and Y. L. Yang, " $\bar{B}_{d,s}^* \rightarrow D^*_{d,s} V$ and $\bar{B}^*_{d,s} \rightarrow D_{d,s} V$ decays in QCD factorization and possible puzzles," *The European Physical Journal C*, vol. 76, no. 10, 2016.
- [73] J. Sun, Y. Yang, N. Wang, Q. Chang, and G. Lu, "Possibility of searching for $B_c^* \rightarrow B_{u,d,s} V, B_{u,d,s} P$ decays," *Physical Review D*, vol. 95, no. 7, article 074032, 2017.
- [74] Q. Chang, L. L. Chen, and S. Xu, "Study of $B_c \rightarrow J/\psi V$ and $B_c^* \rightarrow \eta_c V$ decays within the QCD factorization," *Journal of Physics G: Nuclear and Particle Physics*, vol. 45, no. 7, article 075005, 2018.
- [75] J. Sun, H. Li, Y. Yang, N. Wang, Q. Chang, and G. Lu, " $B^* \rightarrow \bar{D} D$ decays with perturbative QCD approach," *Journal of Physics G: Nuclear and Particle Physics*, vol. 44, no. 7, article 075007, 2017.
- [76] J. Sun, Y. Yang, N. Wang, J. Huang, and Q. Chang, "Study of $B_c^* \rightarrow \psi(1S, 2S)P, \eta_c(1S, 2S)P$ weak decays," *Physical Review D*, vol. 95, no. 3, article 036024, 2017.
- [77] Q. Chang, J. Zhu, N. Wang, and R. M. Wang, "Probing the effects of new physics in decays," *Advances in High Energy Physics*, vol. 2018, Article ID 7231354, 13 pages, 2018.
- [78] J. Zhang, Y. Zhang, Q. Zeng, and R. Sun, "New physics effects of the vector leptoquark on $\bar{B}^* \rightarrow P\tau\bar{\nu}_\tau$ decays," *The European Physical Journal C*, vol. 79, no. 2, 2019.
- [79] J. G. Körner and G. A. Schuler, "Exclusive semi-leptonic decays of bottom mesons in the spectator quark model," *Zeitschrift für Physik C Particles and Fields*, vol. 38, no. 3, pp. 511–518, 1988, Erratum: *Z. Phys. C* 41 (1989) 690.
- [80] J. G. Körner and G. A. Schuler, "Exclusive semileptonic heavy meson decays including lepton mass effects," *Zeitschrift für Physik C: Particles and Fields*, vol. 46, no. 1, pp. 93–109, 1990.
- [81] Y. M. Wang, H. Zou, Z. T. Wei, X. Q. Li, and C. D. Lu, "The transition form factors for semi-leptonic weak decays of J/ψ in QCD sum rules," *The European Physical Journal C*, vol. 54, no. 1, pp. 107–121, 2008.
- [82] Y. L. Shen and Y. M. Wang, " J/ψ weak decays in the covariant light-front quark model," *Physical Review D*, vol. 78, no. 7, article 074012, 2008.
- [83] S. Fajfer, J. F. Kamenik, and I. Nisandzic, " $B \rightarrow D^* \tau \bar{\nu}_\tau$ sensitivity to new physics," *Physical Review D*, vol. 85, no. 9, article 094025, 2012.
- [84] A. Kadeer, J. G. Korner, and U. Moosbrugger, "Helicity analysis of semileptonic hyperon decays including lepton-mass effects," *The European Physical Journal C*, vol. 59, no. 1, pp. 27–47, 2009.
- [85] J. Charles, A. Höcker, H. Lacker et al., "CP violation and the CKM matrix: assessing the impact of the asymmetric B factories," *The European Physical Journal C*, vol. 41, no. 1, pp. 1–131, 2005.
- [86] H. M. Choi, "Decay constants and radiative decays of heavy mesons in light-front quark model," *Physical Review D*, vol. 75, no. 7, article 073016, 2007.
- [87] J. L. Goity and W. Roberts, "Radiative transitions in heavy mesons in a relativistic quark model," *Physical Review D*, vol. 64, no. 9, article 094007, 2001.
- [88] D. Ebert, R. N. Faustov, and V. O. Galkin, "Radiative M1-decays of heavy-light mesons in the relativistic quark model," *Physics Letters B*, vol. 537, no. 3-4, pp. 241–248, 2002.
- [89] S. L. Zhu, Z. S. Yang, and W. Y. P. Hwang, " $D^* \rightarrow D\gamma$ and $B^* \rightarrow B\gamma$ as Derived from QCD Sum Rules," *Modern Physics Letters A*, vol. 12, no. 39, pp. 3027–3035, 1997.
- [90] T. M. Aliev, D. A. Demir, E. Iltan, and N. K. Pak, "Radiative $B^* \rightarrow B\gamma$ and $D^* \rightarrow D\gamma$ decays in light-cone QCD sum rules," *Physical Review D*, vol. 54, no. 1, pp. 857–862, 1996.
- [91] P. Colangelo, F. De Fazio, and G. Nardulli, "Radiative heavy meson transitions," *Physics Letters B*, vol. 316, no. 4, pp. 555–560, 1993.
- [92] C. Y. Cheung and C. W. Hwang, "Strong and radiative decays of heavy mesons in a covariant model," *Journal of High Energy Physics*, vol. 1404, p. 177, 2014.
- [93] W. Jaus, "Covariant analysis of the light-front quark model," *Physical Review D*, vol. 60, no. 5, article 054026, 1999.
- [94] W. Jaus, "Consistent treatment of spin-1 mesons in the light-front quark model," *Physical Review D*, vol. 67, no. 9, article 094010, 2003.
- [95] H. Y. Cheng, C. K. Chua, and C. W. Hwang, "Covariant light-front approach for s-wave and p-wave mesons: its application to decay constants and form factors," *Physical Review D*, vol. 69, no. 7, article 074025, 2004.
- [96] Q. Chang, Y. Zhang, and X. Li, " $\bar{B}^*_{u,d,s} \rightarrow D^*_{u,d,s} V$ ($V = D^{*-}, K^{*-}, \rho^-$) weak decays *," *Chinese Physics C*, vol. 43, no. 10, article 103104, 2019.
- [97] C. Murgui, A. Penuelas, M. Jung, and A. Pich, "Global fit to $b \rightarrow c\tau\nu$ transitions," *Journal of High Energy Physics*, vol. 1909, p. 103, 2019.
- [98] Belle Collaboration <http://arxiv.org/abs/1904.08794>.
- [99] Belle Collaboration <http://arxiv.org/abs/1903.03102>.
- [100] A. K. Alok, D. Kumar, S. Kumbhakar, and S. U. Sankar, " D^* polarization as a probe to discriminate new physics in $\bar{B} \rightarrow D^* \tau \bar{\nu}$," *Physical Review D*, vol. 95, no. 11, article 115038, 2017.
- [101] R. C. Verma, "Decay constants and form factors of s-wave and p-wave mesons in the covariant light-front quark model," *Journal of Physics G: Nuclear and Particle Physics*, vol. 39, no. 2, article 025005, 2012.
- [102] Q. Chang, X. N. Li, X. Q. Li, F. Su, and Y. D. Yang, "Self-consistency and covariance of light-front quark models: testing via P, V , and A meson decay constants, and $P \rightarrow P$ weak transition form factors," *Physical Review D*, vol. 98, no. 11, article 114018, 2018.