

Deep Learning to Find Key Brain Electrode Positions in Emotional EEG

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Abstract

At present, the channels used for EEG acquisition are more than 16, which makes it difficult to wear EEG caps and has poor contact. Therefore, it brings difficulties to the collection of brain waves is not conducive to converting research into applications. It is a well worth studying work in researching how to find the key brain electrode in the existing brain wave, which will greatly reduce the number of EEG acquisition points during application, making it easier to translate the research into practical application. This paper takes emotional EEG as an example to study how to find the key brain electrode points of emotional EEG with deep learning method. Firstly, using the least square regression algorithm to calculate the characteristic coefficients of each electrode point; secondly, according to the law of the characteristic coefficient value, grouping the key EEG poles for experiment. In the grouping experiment, the Conv1d-GRU model used to train and verify the EEG data of the corresponding electrode points. Finally, from the results of various grouping experiments, it concluded that the selection method of the key EEG level points should be the electrode points with positive characteristic coefficient, and the accuracy of verification is 97.6%. With experiments, it confirmed that there are key electrode points in the detection of emotional EEG by 16-channel OpenBCI. There are only six key electrode points of emotional EEG; that is to say, the EEG data collected by only six key electrode points can identify seven kinds of emotional EEG.

Keywords

Electroencephalogram, Least Squares Algorithm, Deep Learning, Emotion Recognition

1. Introduction

There are 128 electrodes of EEG. The more collection points, the larger the

amount of data collected. More electrode points mean more electrodes need to be connected, which adds difficulties to the acquisition work. In the previous research work, the 16-channel EEG cap used to collect information. During the process of collecting information, several problems occurred. The electrode cap was loosened a little, which would lead to poor contact, and the cap was tightened a little, which would lead to scalp tingling. Adjusting the EEG cap during the whole acquisition process required help and took a long time, and the experimenter was very painful.

Too many collection points are not conducive to research conversion, as different people have different head shapes, and collection points need to attach to the scalp closely. The fewer collection points, the more conducive it is to research conversion.

In the previous research work, the main contributions are the Conv1d-GRU EEG recognition model and the method of action and idea EEG acquisition [1] [2]. In paper [1], the research content is to use Conv1d-GRU model to identify emotional EEG types. The experimental results show that Conv1d-GRU model has a high accuracy in identifying EEG. In paper [2], the content of the study is how to do the action idea EEG, and use the Conv1d-GRU model to identify, the experimental results show that the action idea EEG should be under the static state of the human hand.

In the experiment of seven kinds of emotion EEG recognition, both the 8-channel experiment and the 16-channel experiment can effectively identify emotion types. So can we reduce some channels to recognize emotion? Therefore, this paper studies the key electrodes of EEG in emotion recognition.

Searching the keyword “key EEG electrodes” on CNKI, there are no references. Searching “Key EEG electrode points” in IEEE explore Digital Library, there are 32 records of relevant results, but there is no connection between the 32 records and Key EEG electrode points, most of them are EEG characteristics, for example, references [3] [4] [5]. In paper [6], the author studied the EEG during driving and established a data set to detect fatigue driving. At the same time, the author also mentioned that the presence of key EEG electrodes and excessive EEG channels in the EEG would introduce noise and increase the computational burden. In paper [7], the author studied the effects of different acupuncture manipulations through EEG. Paper [8] is about electrode modeling, and there is a paper about the correlation between EEG network and influencing factors [9]. There is a big difference between other papers and this paper. Therefore, it is a creative idea to study the key electrodes of EEG.

2. Related Work

2.1. Characteristics of Least Squares Method

An EEG collects many multi-dimensional signals, which are non-linearly. Calculating the characteristic coefficients of EEG signals need Least Squares Regression (LSR).

The least squares method is a mathematical optimization technique. It seeks the best function match for the data by minimizing the sum of squared errors. The least squares method can be used to easily obtain unknown data, and the sum of squares of the errors between the obtained results and the actual value data is the smallest [10]. It has the following characteristics.

1) Linear characteristic

The estimators are respectively linear function of sample observations,
 $y_1 = b_1 * X_1 + b_2 * X_2 + \dots + b_n * X_n$.

2) Unbiased

The expected value of the parameter estimator corresponds to the overall true parameters.

3) Minimum variance

Compared to any linear unbiased estimator obtained by other methods, the least squares algorithm is the best estimator.

The feature of emotional EEG is a temporal data collection of multiple electrode points at each moment, at each time point, 16 channels of emotional EEG waves collected, with each channel corresponding to one data. How to find which channels of data are critical in the collected time series dataset, the linear regression problems in artificial intelligence can solve that question. Treat the collected electrode data as X , the X is an $n * 16$ vector. The action idea type is treated as Y , the Y is an $n * 1$ vector. Solve for B in $Y = XB$, the B is a vector of $16 * 1$. The problem of the importance of data at each electrode point can transform into the problem of finding the data coefficients of the electrode points, and further transformed into a parameter vector for minimizing the sum of squares of residuals in multiple linear regression. Currently, the commonly used method to solve this problem is the least squares method [11] [12] [13] [14] [15].

The least square has the minimum variance, linearity and unbiased of the estimator, and is one of the effective algorithms in the field of machine learning. According to the sample data, using the least square estimator to obtain the estimator of the parameters of the simple linear regression model [16].

2.2. The Principle of Least Square Method Applied to EEG

We regard the emotional EEG data at a certain time as X vector, the emotional classification at this time as Y vector, and the data coefficient of each electric pole as B vector. Then the formula of the multivariate linear model is as follow 1.

$$h(x) = \sum_{i=1}^n b_i x_i + e \quad (1)$$

In Formula (1), $\sum_{i=1}^n b_i x_i$ represents the sum of the values collected by all EEG electrodes multiplied by the response coefficient. We will record it as Y , which is the real value, $h(x)$ is the calculated value, and e is the difference between the real value and the calculated value.

Suppose there are m samples. For m samples, there are the following linear simultaneous Equations (2)

$$\begin{cases} b_1x_{11} + b_2x_{12} + b_3x_{13} + \cdots + b_nx_{1n} = y_1 \\ b_1x_{21} + b_2x_{22} + b_3x_{23} + \cdots + b_nx_{2n} = y_2 \\ \vdots \\ b_1x_{m1} + b_2x_{m2} + b_3x_{m3} + \cdots + b_nx_{mn} = y_m \end{cases} \quad (2)$$

In linear simultaneous Equations (2), x_{ij} represents the data collected by the j th EEG pole in the i th sample, and y_i represents the emotion type corresponding to the i th sample. Convert the linear simultaneous equations into a matrix to obtain the expression 3

$$\begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} \quad (3)$$

The values of x and y in expression 3 are known, and the value of b needs to be solved. Simplify expression 3 to expression 4

$$XB = Y \quad (4)$$

In expression 4, X is the data matrix collected by EEG poles, Y is the emotion type matrix corresponding to X , and B is the coefficient matrix. Get the expression 5 according to the least square method.

$$E(b) = \sum_{i=1}^m (\hat{y}_i - y_i)^2 \quad (5)$$

In expression 5, \hat{y}_i is the calculated value, and y_i is the true value. According to Formulas (4) and (5), get expression 6.

$$E(b) = \|XB - Y\|_2^2 \quad (6)$$

In expression 6, XB is the calculated \hat{Y} and Y is the true value. According to the norm operation rules, expression (6) expanded to obtain expression 7

$$\|XB - Y\|_2^2 = \text{tr}((XB - Y)^T (XB - Y)) \quad (7)$$

Take $J(B) = (XB - Y)^T (XB - Y)$ and expand it to get Formula (8)

$$J(B) = B^T X^T XB - 2(XB)^T Y + Y^T Y \quad (8)$$

According to the rule of least square method, $J(B)$ takes the partial derivative of B , and the value of $J(B)$ is the minimum when the derivative is zero. At this time, the real value is the closest to the calculated value. The expression for $J(B)$ to find the partial derivative of B is Formula (9)

$$\frac{\partial J(B)}{\partial B} = \frac{\partial (B^T X^T XB - 2(XB)^T Y + Y^T Y)}{\partial B} \quad (9)$$

Since Equation (10) and Equation (11), Equation (9) can derive Equation (12)

$$\frac{\partial (B^T XB)}{\partial B} = XB + X^T B \quad (10)$$

$$\frac{\partial (B^T X)}{\partial B} = \frac{\partial (X^T B)}{\partial B} = X \quad (11)$$

$$\frac{\partial J(B)}{\partial B} = 2X^T XB - 2X^T Y \quad (12)$$

It is known from Equation (12) that when $\frac{\partial J(B)}{\partial B} = 0$, the sum of squares of residuals is the minimum, and then Equation (13) is obtained

$$X^T XB = X^T Y \quad (13)$$

Calculate the coefficient B vector matrix by Formula (13), as shown in Equation (14)

$$B = (X^T X)^{-1} X^T Y \quad (14)$$

In Equation (14), X is the data matrix collected by the electrode of EEG, Y is the vector matrix of emotion category, and B is the coefficient vector matrix of X , representing the relationship between EEG electrodes and emotion classification.

2.3. Conv1d-GRU Introduction

The Conv1d-GRU model plays an important role in AIEEG recognition. Indeed, the Conv1d-GRU model was proposed in our previous work, and it played a very important role, as it can quickly verify whether selecting key values is effective.

In the previous research work, in order to identify the EEG classification, the Conv1d-GRU model was proposed in the paper [1] [2]. The Conv1d-GRU model has a high recognition rate, and the data transmitted into the model training using the original data to reduce the data processing time. This paper still uses this recognition model in the experiment, and its model structure shown in **Figure 1**.

The input layer will change according to the number n of channels. The “ m ” is the size of convolution kernel. There are n cores of $m * 1$ in Conv1D. The input data in the EEG emotion classification model can fully extract the features in the sequence after two Conv1D convolutions. In order to avoid gradient disappearance, BN (Batch Normalization) layer added after two Conv1D operations. Because BN inhibits gradient disappearance, the depth network can use sigmoid and tanh excitation functions after the sequence passes through the BN layer. At the same time, BN has a regularization effect, so it does not need to rely on the dropout and can reduce over fitting.

The GRU has three layers. The first and second layer of GRU return all the hidden states of the last layer, and the third layer returns the hidden state of the last step of the last layer. Finally, the dense layer executed, the GRU output sent to the full-connected neural network, and the final classification obtained with the softmax excitation function.

3. Characteristic Coefficients of EEG Signals

EEG data can calculate coefficient vector matrix with **Figure 2**.

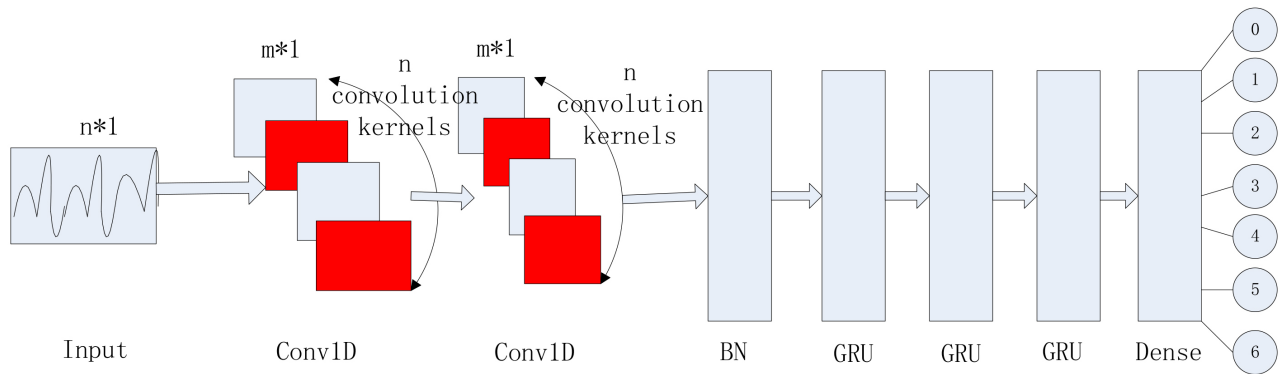


Figure 1. Conv1d-GRU model.

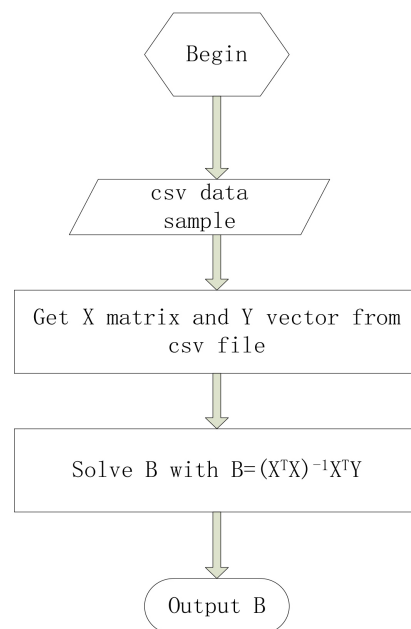


Figure 2. Flow of coefficient vector matrix B.

In this experiment, the open source brain-computer interfacing (OpenBCI) equipment used to collect EEG, and the coefficient matrix solved by using 16 channel emotional EEG data. With the sample size of 5376, according to the calculation formula of the least square method to solve the coefficient vector matrix B, the results as shown in **Figure 3**.

According to the characteristics of the coefficient, the larger the coefficient of the variable is, the more important the variable is. According to the results in **Figure 3**, the goal of this experiment is to find as few electrodes as possible to determine the emotional classification. For this reason, the adopted scheme is—at firstly, a little more electrode points, and then gradually reduces the electrode points. Therefore, the scheme divided into four schemes: full positive coefficient, maximum coefficient of absolute value, and coefficient of both positive and maximum absolute value. The 8-channel and 16-channel have the same electrode points, and those electrode points are in the previous effective scheme.

$$B = \begin{bmatrix} 1.50766328e-04 & -1.66176520e-04 & -2.28309952e-04 & -1.89351663e-05 \\ -1.28105236e-05 & -2.80023141e-06 & -1.33482345e-05 & -2.81368615e-06 \\ -1.18583509e-04 & -1.96728113e-05 & 3.38727621e-05 & 2.02137339e-05 \\ 1.01649441e-05 & -1.56769265e-05 & 1.12569133e-04 & 3.45256453e-05 \end{bmatrix}$$

Figure 3. Coefficient vector matrix B.

4. Experimental Scheme

According to the previous analysis, the experiment conducted in four situations. The experimental method is to send the data of the corresponding electrode points to the Conv1d-GRU model for training and testing, and judge whether the current combination of electrode points is effective according to the accuracy of the test.

4.1. Take All Positive Coefficients

According to the value of coefficient matrix B, the EEG electrode positions corresponding to all positive values are 1, 11, 12, 13, 15, and 16. After 200 epochs of training with Conv1d-GRU model, the change of accuracy rate shown in **Figure 4**, the change of loss rate shown in **Figure 5**, and the final prediction result shown in **Figure 6**.

From **Figure 4** and **Figure 5**, it shows that the model training speed is very fast, and the accuracy can reach about 98%.

From **Figure 6**, it shows that the accuracy of the prediction results is 97.6%, which is very high. That means that the electrode positions of 1, 11, 12, 13, 15, and 16 EEG can completely complete the EEG emotion experiment, and its accuracy can reach the effect of 16-channel.

4.2. Take the Coefficient with Large Absolute Value

According to the value of coefficient matrix B, take the coefficient with the absolute value of e^{-4} and the corresponding EEG electrode position is 1, 2, 3, 9, and 15.

Through 200 epochs of Conv1d-GRU model training, the change of accuracy rate is shown in **Figure 7**, the change of loss rate is shown in **Figure 8**, and the final prediction result is shown in **Figure 9**.

During the experiment, the improvement of accuracy rate was very slow, and it was difficult to improve the accuracy rate. From the final loss rate, it shows that the loss data remained within a fluctuation range.

From **Figure 9**, it shows that the accuracy rate of prediction results can only reach 75%, which is very low. That means that the five EEG pole positions 1, 2, 3, 9 and 15 cannot complete the EEG emotion experiment.

4.3. Positive Value and Large Coefficient

According to the value of coefficient matrix B, the value is the large positive coefficient of e^{-4} , and the corresponding EEG electrode position is 1 and 15.

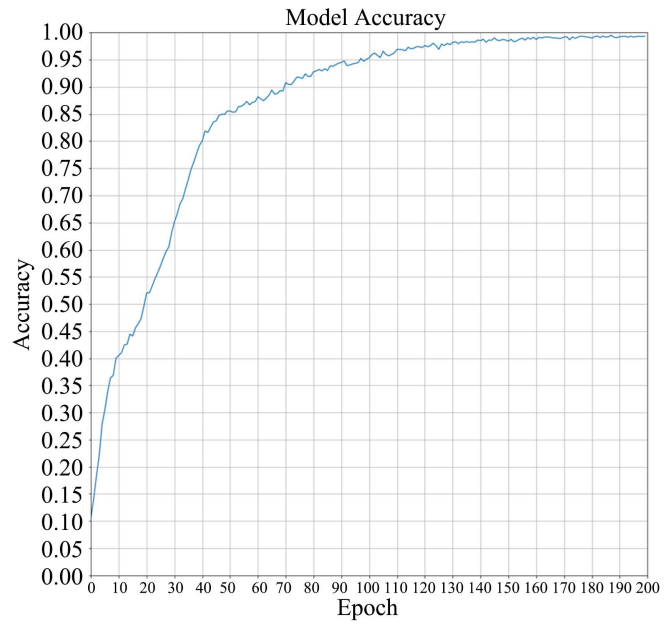


Figure 4. Accuracy change curve of model 1.

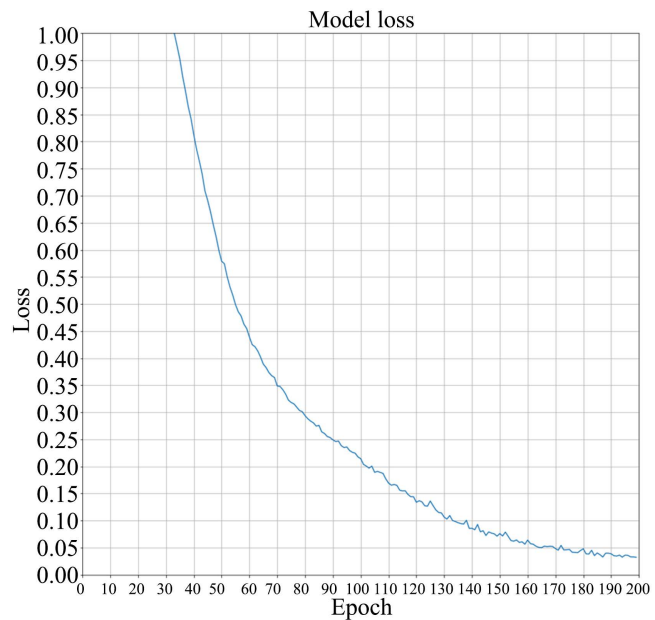


Figure 5. Loss rate change curve of model 1.

```
[24]: 1 model.load_weights("./depChannel.h5")

[25]: 1 os.environ['CUDA_VISIBLE_DEVICES'] = '-1'
      2 result = model.evaluate(wave_test, target_test, verbose=0)
      3 print(model.metrics_names)
      4 print(result[0])
      5
      6

['loss', 'accuracy']
0.0238339499628002
```

Figure 6. Prediction results of model 1.

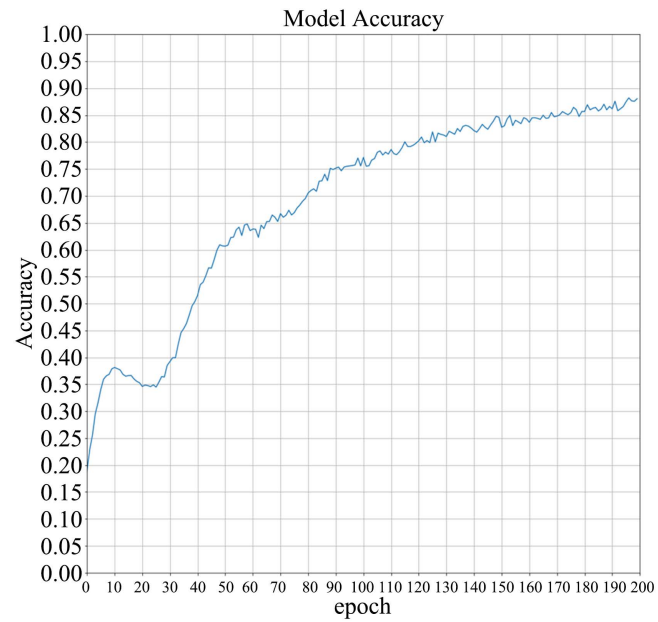


Figure 7. Accuracy change curve of model 2.

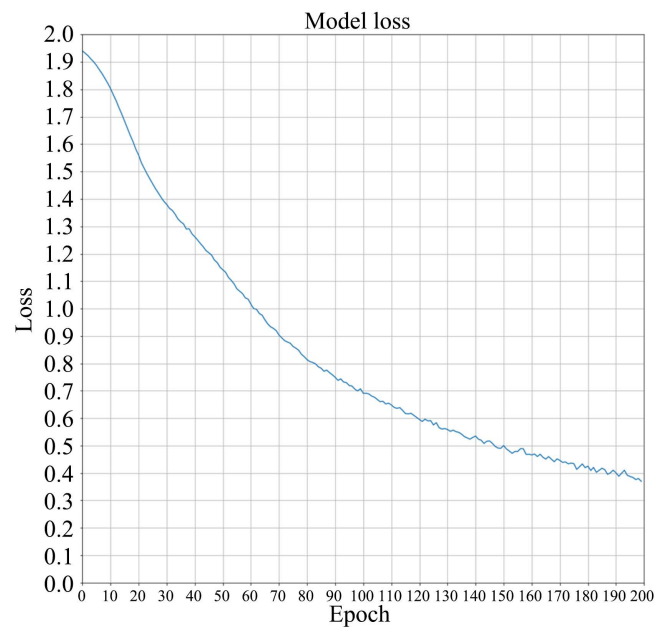


Figure 8. Loss rate change curve of model 2.

```
[64]: 1 model.load_weights("./depChannelModel2.h5")

[65]: 1 os.environ['CUDA_VISIBLE_DEVICES'] = '-1'
      2 result = model.evaluate(wave_test, target_test, verbose=0)
      3 print(model.metrics_names)
      4 print(result[0])
      5
      6

['loss', 'accuracy']
0.24929416155815123
```

Figure 9. Prediction results of model 2.

Through 500 epochs of Conv1d-GRU model training, the change of accuracy rate shown in **Figure 10**, the change of loss rate shown in **Figure 11**, and the final prediction result shown in **Figure 12**.

From **Figure 10** and **Figure 11**, it shows that the accuracy rate cannot be improved, so EEG emotion cannot be recognized.

From **Figure 12**, it shows that only 1 and 15 electrodes of EEG cannot complete the prediction.

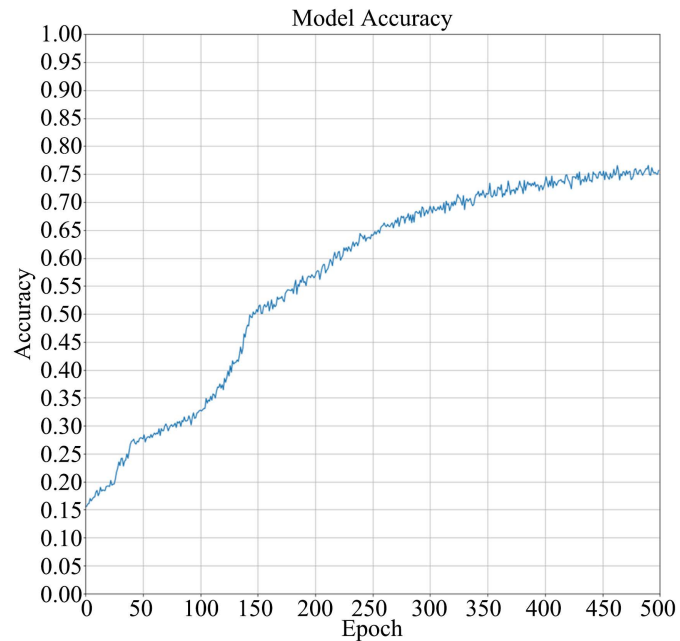


Figure 10. Accuracy change curve of model 3.

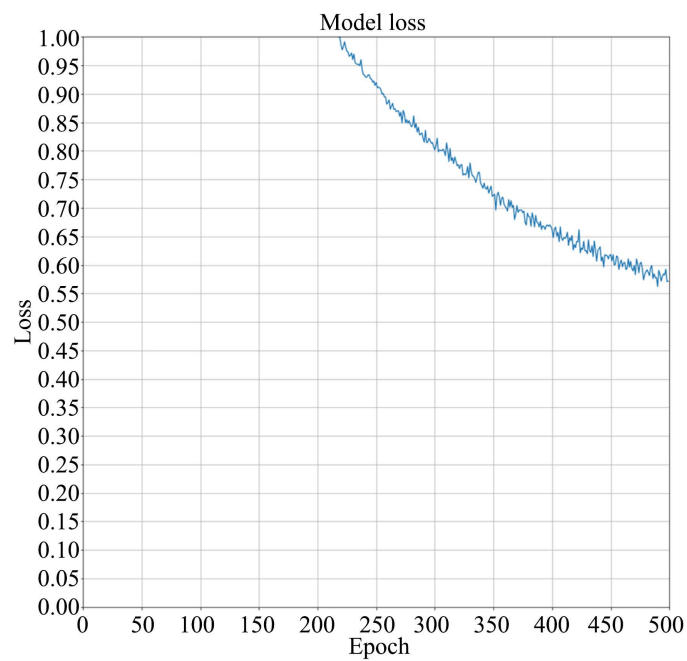


Figure 11. Loss rate change curve of model 3.

```
[20]: 1 model.load_weights("./depChannelModel3.h5")

[22]: 1 os.environ['CUDA_VISIBLE_DEVICES'] = '-1'
      2 result = model.evaluate(wave_test, target_test, verbose=0)
      3 print(model.metrics_names)
      4 print(result[0])
      5
      6

['loss', 'accuracy']
1.0232756850502707
```

Figure 12. Prediction results of model 3.

4.4. EEG Electrode Points in Both Integrated 8-Channel and Scheme 4.1

According to the EEG electrode location map provided by OpenBCI, Select common EEG poles in 8-channel and Scheme 4.1. The electrodes of EEG meeting the conditions are 1, 11, 12 in 16-channel, corresponding to the 1, 3 and 4 EEG poles of 8-channel.

Through 500 epochs of Conv1d-GRU model training, the change of accuracy rate shown in **Figure 13**, the change of loss rate shown in **Figure 14**, and the final prediction result is shown in **Figure 15**.

From **Figure 13** and **Figure 14**, it shows that the training time of the model is a little long, but it keeps rising, and finally can reach more than 88%, indicating that this combination has certain prediction ability. That is to say, emotional EEG waves are mainly at positions 1, 11 and 12 of EEG electrodes in 16-channel, corresponding to the 1, 3 and 4 EEG poles of 8-channel.

The prediction accuracy rate is 89.7%, which means that when emotional EEG recognition is not very accurate, only electrode points 1, 11 and 12 can test EEG, greatly reducing the data and location of EEG acquisition.

5. Verify with 8-Channel Experimental Data

According to the experimental scheme in 4.4, it is believed that when emotional EEG is collected, only three positions 1, 11 and 12 of 16-channel need to be collected, which correspond to positions 1, 3 and 4 of 8-channel EEG electrodes.

According to this conclusion, the data of channels 1, 3 and 4 of the 8-channel emotional EEG experimental data selected for training and testing. The change of accuracy rate is shown in **Figure 16**, the change of loss rate is shown in **Figure 17**, and the final prediction result is shown in **Figure 18**.

From **Figure 16** and **Figure 17**, it shows that the recognition rate of emotional EEG by selecting channel 1, 3 and 4 data from 8-channel is about 88%. The experimental results further prove that the main electrode points of emotional EEG are at positions 1, 11 and 12 of 16-channel, corresponding to positions 1, 3 and 4 of 8-channel.

From **Figure 18**, it shows that the accuracy rate is 86.2%, and the key acquisition points are channels 1, 3 and 4 of 8-channel, corresponding to channels 1, 11 and 12 of 16-channel.

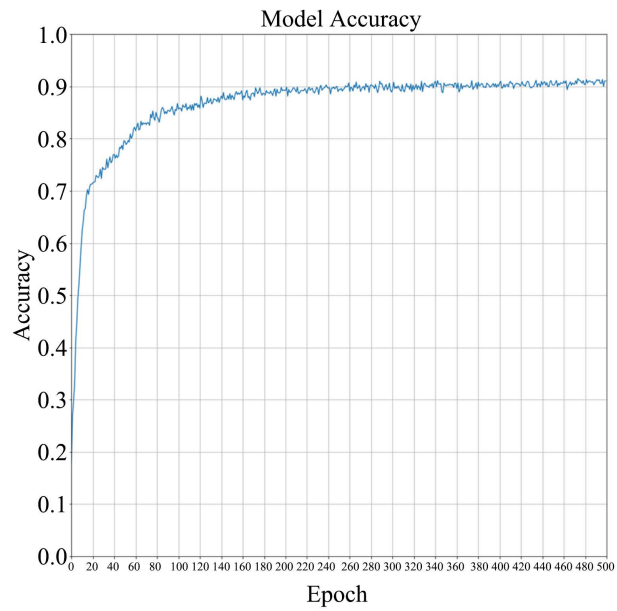


Figure 13. Accuracy change curve of model 4.

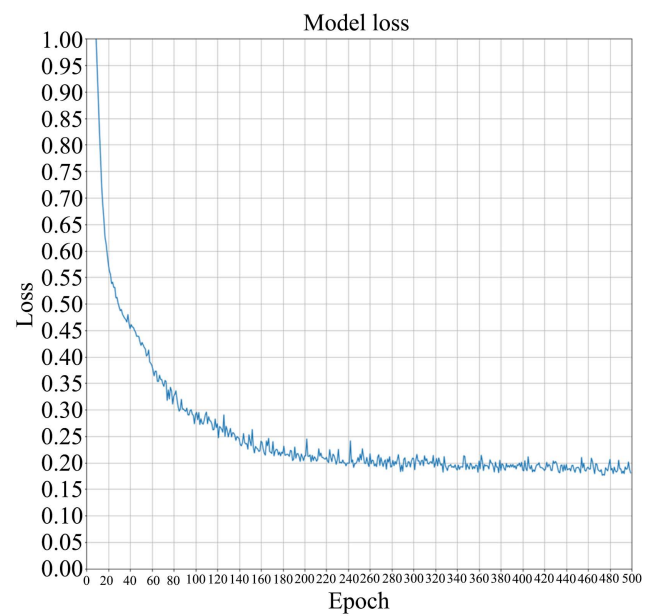


Figure 14. Loss rate change curve of model 4.

```

1 import os
1 model.load_weights("./depChannelModel4.h5")
1 os.environ['CUDA_VISIBLE_DEVICES'] = '-1'
2 result = model.evaluate(wave_test, target_test, verbose=0)
3 print(model.metrics_names)
4 print(result[0])

['loss', 'accuracy']
0.10267671851136467

```

Figure 15. Prediction results of model 4.

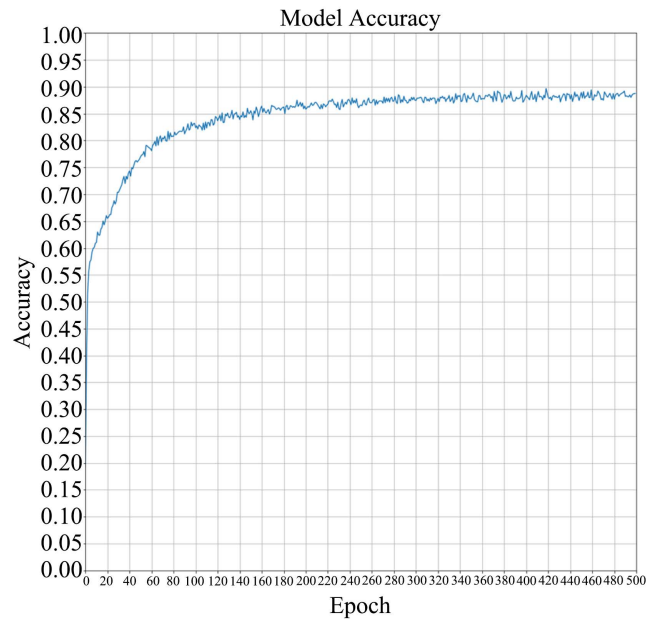


Figure 16. Accuracy change curve of model 5.

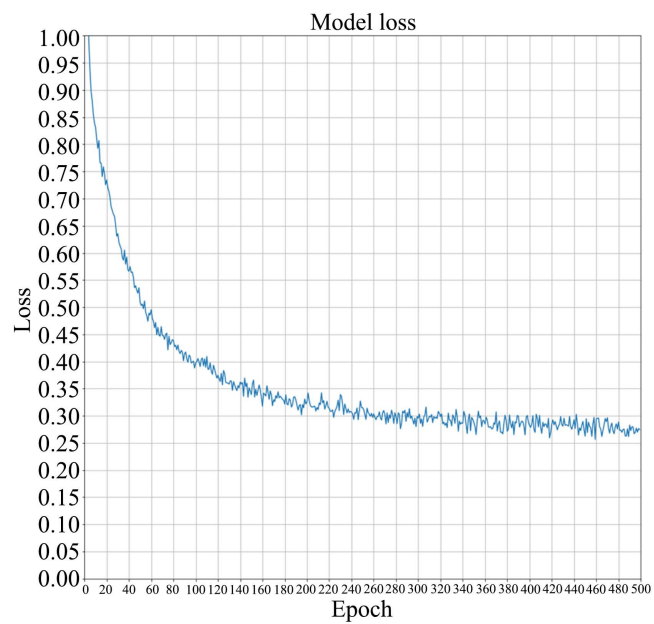


Figure 17. Loss rate change curve of model 5.

```
[16]: 1 model.load_weights("./depChannelModel15.h5")

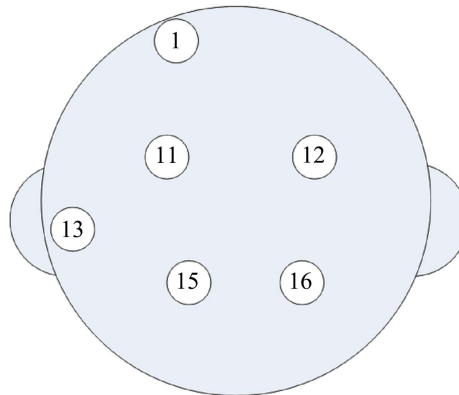
[17]: 1 os.environ['CUDA_VISIBLE_DEVICES'] = '-1'
      2 result = model.evaluate(wave_test, target_test, verbose=0)
      3 print(model.metrics_names)
      4 print(result[0])
      5
      6

['loss', 'accuracy']
0.13775483811540262
```

Figure 18. Prediction results of model 5.

Table 1. Comparison of experiment result.

SN	Grouping Type	Verification Accuracy
1	positive coefficients	97.6%
2	larger absolute coefficient	75%
3	positive and larger coefficients	0%
4	positive coefficients are in both 8-channel and 16-channel	86.2%

**Figure 19.** The key EEG potentials of emotional EEG.

6. Results

In this paper, the linear regression method and deep learning model in artificial intelligence used to calculate and verify the electrode correlation of emotional brainwaves.

The result of group comparison experiment is as **Table 1**.

From **Table 1**, it shows that when selecting the electrode point represented by the positive correlation coefficient, the verification accuracy is 97.6%; In that case, only 6 electrode points can be collected during the application of emotional EEG. When emotional EEG is applied in applications with low accuracy requirements, such as 86.2%, the emotional EEG only 3 electrode points can be collected.

7. Conclusions

It is a very bad experience to feel too many electrodes doing EEG experiment with the EEG cap. This paper uses seven kinds of emotional EEG waves to find out the key EEG potentials of emotional EEG waves, as shown in **Figure 19**.

In **Figure 19**, the 6 electrode points can identify seven emotional EEG waves and the verification accuracy is 97.6%.

Through the research in this paper, we can confirm that there are key brain potentials in EEG waves. We can develop many applications targeting these potentials, using the least number of touchpoints, such as learning efficiency monitoring and improvement, depression monitoring and improvement, etc.

Acknowledgements

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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