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Exact Traveling Wave Solutions of Nonlinear Evolution Equation via Enhanced (G'/G)-Expansion Method

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Abstract

In this work, we employ an enhanced (G'/G)-expansion method to study the nonlinear evolution equations (NLEEs). Here we derive exact traveling wave solutions for the modified Burgers-KDV equation. The obtained results show that the applied equation reveals richness of explicit soliton and periodic solutions. It has been shown that the enhanced (G'/G)-expansion method is effective and can be used for many other NLEEs in mathematical physics.

Keywords: Enhanced (G'/G)-expansion method; Modified Burgers-KDV equation; Traveling waves; soliton; Exact solution; NLEEs.

1. Introduction

NLEEs are encountered in various fields of mathematics, physics, chemistry, biology, engineering and numerous applications. Exact solutions of NLEEs play an important role in the proper understanding of qualitative features of many phenomena and processes in various areas of natural science. Exact solutions of nonlinear equations graphically demonstrate and allow unscrambling the mechanisms of many complex nonlinear phenomena such as spatial localization of transfer processes, multiplicity or absence steady states under various conditions, existence of peaking regimes and many others. Even those special exact solutions that do not have a clear physical meaning can be used as test problems to verify the consistency and estimate errors of various numerical, asymptotic, and approximate analytical methods. Exact solutions can serve as a basis for perfecting and testing computer algebra software packages for solving NLEEs. It is significant that many equations of physics, chemistry, and biology contain empirical parameters or empirical functions. Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore,

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investigating exact traveling wave solutions is becoming successively attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as, the Hirota's bilinear transformation method [1,2], the modified simple equation method [3-7], the tanh-function method [8,9], the Exp-function method [10-13], the Jacobi elliptic function method [14], the (G'/G)-expansion method and its various extensions [15-32], the homotopy perturbation method [33,34], the transformed rational function method [35,36], the multiple exp-function method [37,38], the generalize Hirota bilinear method [39], the Simplest equation method and its various form [40-44], the trigonometric function series method [45, 46], the modified mapping method and extended mapping method [47], the bifurcation method and qualitative theory of dynamical system [48], the dynamical systems approach [49], the auxiliary ordinary differential equation method [50], the tanh-coth Expansion Method and Jacobi Elliptic Function Expansion Method [51], the infinite series method [52] and so on.

Various ansatze have been proposed for seeking traveling wave solutions of nonlinear differential equations. The choice of an appropriate ansätze is of great importance when using the direct methods.

Recently, Wang et. al [15] have introduced a simple method which is called the (G'/G)-expansion method to look for traveling wave solutions of nonlinear evolution equations, where $G = G(\xi)$ satisfies the second order linear ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu\, G(\xi) = 0$,where λ and μ are arbitrary constants and $u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \cdots\cdots\cdots$ be the traveling wave solution of NLEEs. By means of this method they have solved the KdV equation, the mKdV equation, the variant Boussinesq equations and the Hirota–Satsuma equations.

Guoet. al [17] have introduced an another method so called extended (G'/G)-expansion method where $G = G(\xi)$ satisfies the second order linear ordinary differential equation:

$G'' + \mu G = 0$, where $G' = \dfrac{dG(\xi)}{d\xi}$, $G'' = \dfrac{d^2 G(\xi)}{d\xi^2}$, $\xi = x - Vt$, V is a constant and $u(\xi) = a_0 + \sum_{i=1}^{n}\left(a_i (G'/G)^i + b_i (G'/G)^{i-1}\sqrt{\sigma\left(1 + \frac{(G'/G)^2}{\mu}\right)}\right)$ be the traveling wave solution. They proposed extended (G'/G)-expansion method to construct travelling wave solutions of Whitham–Broer–Kaup–Like equations and coupled Hirota–Satsuma KdV equations. For further references of the (G'/G)- expansion method see the articles [15-32].

The objective of this article is to present an enhanced (G'/G)-expansion method to construct the exact solitary wave solutions for NLEEs in mathematical physics via the Modified Burgers-KDV equation.

The article is arranged as follows: In section 2, the enhanced (G'/G)-expansion method is discussed. In section 3, we apply this method to the nonlinear evolution equations pointed out above; in section 4, results and discussion; and in section 5 conclusions are given.

2. Methodology

In this section, we describe the enhanced (G'/G)-expansion method for finding traveling wave solutions of NLEEs. Suppose that a nonlinear partial differential equation, say in two independent variables x and t is given by

$$\Re(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \cdots\cdots\cdots\cdots) = 0, \tag{2.1}$$

where $u(\xi) = u(x,t)$ is an unknown function, $\Re$ is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [28,29]:

Step 1. Combining the independent variables x and t into one variable $\xi = x \pm V t$, we suppose that

$$u(\xi) = u(x,t), \qquad \xi = x \pm V t, \tag{2.2}$$

where wave speed $V \in R - \{0\}$.

The traveling wave transformation Eq. (2.2) permits us to reduce Eq. (2.1) to the following ODE:

$$F(u, u', u'', \cdots\cdots\cdots) = 0, \tag{2.3}$$

where F is a polynomial in $u(\xi)$ and its derivatives, while $u'(\xi) = \dfrac{du}{d\xi}$, $u''(\xi) = \dfrac{d^2u}{d\xi^2}$ and so on.

Step 2.We suppose that Eq.(2.3) has the formal solution

$$u(\xi) = \sum_{i=-n}^{n} \left(\frac{a_i (G'/G)^i}{\left(1 + \lambda (G'/G)\right)^i} + b_i (G'/G)^{i-1} \sqrt{\sigma \left(1 + \frac{(G'/G)^2}{\mu}\right)} \right), \tag{2.4}$$

where $G = G(\xi)$ satisfy the equation $G'' + \mu G = 0$, (2.5)

in which $a_i, b_i \ (-n \le i \le n; n \in \mathrm{N})$ and λ are constants to be determined later, and $\sigma = \pm 1, \mu \ne 0$.

Step 3. The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq.(2.1) or Eq.(2.3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = n$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s(n+q)\,. \tag{2.6}$$

Therefore we can find the value of n in Eq.(2.4), using Eq.(2.6).

Step 4. We substitute Eq. (2.4) into Eq.(2.3) using Eq. (2.5) and then collect all terms of same powers of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}$ together, then set each coefficient of them to zero to yield an over-determined system of algebraic equations, solve this system for a_i, b_i, λ and V.

Step 5. From the general solution of Eq. (2.5), we get

When $\mu < 0$,

$$\frac{G'}{G} = \sqrt{-\mu}\tanh(A + \sqrt{-\mu}\xi) \tag{2.7}$$

And $$\frac{G'}{G} = \sqrt{-\mu}\coth(A + \sqrt{-\mu}\xi) \tag{2.8}$$

Again, when $\mu > 0$,

$$\frac{G'}{G} = \sqrt{\mu}\tan(A - \sqrt{\mu}\xi) \tag{2.9}$$

And $$\frac{G'}{G} = \sqrt{\mu}\cot(A + \sqrt{\mu}\xi) \tag{2.10}$$

where A is an arbitrary constant. Finally, substituting $a_i, b_i\,(-n \le i \le n; n \in \mathrm{N})$, λ, V and Eqs. (2.7)-(2.10) into Eq. (2.4) we obtain traveling wave solutions of Eq. (2.1).

3. Application

In this section, we will exert enhanced (G'/G)-expansion method to solve the Modified Burgers-KDV equation in the form,

$$u_t + pu^2u_x + qu_{xx} - ru_{xxx} = 0\,, \tag{3.1}$$

where p, q, r are nonzero constants and $u(x,t)$ is the amplitude of the relative wave mode.

The traveling wave transformation equation $u(x,t) = u(\xi)$, $\xi = x - Vt$ transform Eq.(3.1) to the following ordinary differential equation:

$$-Vu' + pu^2u' + qu'' - ru''' = 0. \tag{3.2}$$

On integrating Eq. (3.2) with respect to ξ once, we have

$$C - Vu + \frac{1}{3}pu^3 + qu' - ru'' = 0, \tag{3.3}$$

where C is a constant of integration. The homogeneous balance between the highest-order derivative u'' and the nonlinear term u^3 from Eq. (3.3) yields $n = 1$.

Therefore $n = 1$ reduces Eq. (2.4) to

$$\begin{aligned} u(\xi) = {} & \frac{a_{-1}(1+\lambda(G'/G))}{(G'/G)} + a_0 + \frac{a_1(G'/G)}{1+\lambda(G'/G)} + b_{-1}(G'/G)^{-2}\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)} \\ & + b_0(G'/G)^{-1}\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)} + b_1\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)} \end{aligned}, \tag{3.4}$$

where $G = G(\xi)$ satisfies Eq.(2.5). Substitute Eq.(3.4) along with Eq.(2.5) into Eq.(3.3). As a result of this substitution, we get a polynomial of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}$.

From these polynomials, we equate the coefficients of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{1}{\mu}(G'/G)^2\right)}$, and setting them to zero, we get an over-determined system that consists of twenty five algebraic equations. Solving these algebraic equations for a_i, b_i, λ and V, with the aid of symbolic computer software Maple, we have the following seven sets of results:

Set 1: $C = \mp\frac{\sqrt{6}}{54}\left(\frac{\left(q^2+144r^2\mu\right)q}{r\sqrt{pr}}\right), V = \frac{1}{6r}\left(q^2-48r^2\mu\right), \lambda = 0, a_{-1} = \pm\mu\sqrt{\left(\frac{6r}{p}\right)},$

$$a_0 = \mp \frac{q}{\sqrt{6pr}}, a_1 = \mp \sqrt{\left(\frac{6r}{p}\right)}, b_{-1} = 0, b_0 = 0, b_1 = 0.$$

Set 2: $C = \pm \frac{\sqrt{6}}{54}\left(\frac{\left(q^2 + 36r^2\mu\right)q}{r\sqrt{pr}}\right), V = \frac{1}{6r}\left(q^2 - 12r^2\mu\right), \lambda = 0, a_{-1} = 0,$

$$a_0 = \pm \frac{q}{\sqrt{6pr}}, a_1 = \pm \sqrt{\left(\frac{6r}{p}\right)}, b_{-1} = 0, b_0 = 0, b_1 = 0.$$

Set 3 : $C = \pm \frac{\sqrt{6}}{54}\left(\frac{\left(q^2 + 9r^2\mu\right)q}{r\sqrt{pr}}\right), V = \frac{1}{6r}\left(q^2 - 3r^2\mu\right), \lambda = 0, a_{-1} = 0,$

$$a_0 = \pm \frac{q}{\sqrt{6pr}}, a_1 = \pm \frac{1}{2}\sqrt{\left(\frac{6r}{p}\right)}, b_{-1} = 0, b_0 = 0, b_1 = \pm \frac{1}{2}\sqrt{\left(\frac{6r\mu}{p\sigma}\right)}.$$

Set 4: $C = \mp \frac{\sqrt{6}}{54}\left(\frac{\left(q^2 + 36r^2\mu\right)q}{r\sqrt{pr}}\right), V = \frac{1}{6r}\left(q^2 - 12r^2\mu\right), \lambda = \lambda, a_{-1} = \pm \mu\sqrt{\left(\frac{6r}{p}\right)},$

$$a_0 = \mp \frac{\left(q + 6r\mu\lambda\right)}{\sqrt{6pr}}, a_1 = 0, b_{-1} = 0, b_0 = 0, b_1 = 0.$$

Set 5: $C = \pm \frac{\sqrt{6}}{54}\left(\frac{\left(q^2 + 36r^2\mu\right)q}{r\sqrt{pr}}\right), V = \frac{1}{6r}\left(q^2 - 12r^2\mu\right), \lambda = \lambda, a_{-1} = 0,$

$$a_0 = \mp \frac{\left(6r\mu\lambda - q\right)}{\sqrt{6pr}}, a_1 = \pm \sqrt{\left(\frac{6r}{p}\right)}\left(\mu\lambda^2 + 1\right), b_{-1} = 0, b_0 = 0, b_1 = 0.$$

Set 6: $C = \mp \frac{\sqrt{6}}{54}\left(\frac{\left(q^2 + 9r^2\mu\right)q}{r\sqrt{pr}}\right), V = \frac{1}{6r}\left(q^2 - 3r^2\mu\right), \lambda = \lambda, a_{-1} = \pm \mu\sqrt{\left(\frac{6r}{p}\right)},$

$$a_0 = \mp \frac{\left(q + 3r\mu\lambda\right)}{\sqrt{6pr}}, a_1 = 0, b_{-1} = 0, b_0 = \pm \frac{\mu}{2}\sqrt{\left(\frac{6r}{p}\right)}, b_1 = 0.$$

Set 7: $C = \mp\frac{\sqrt{6}}{54}\left(\frac{\left(q^2 + 9r^2\mu\right)q}{r\sqrt{pr}}\right), V = \frac{1}{6r}\left(q^2 - 3r^2\mu\right), \lambda = -\frac{q}{3r\mu}, a_{-1} = \pm\frac{\mu}{2}\sqrt{\left(\frac{6r}{p}\right)},$

$$a_0 = 0, a_1 = 0, b_{-1} = 0, b_0 = \pm\frac{\mu}{2}\sqrt{\left(\frac{6r}{p\sigma}\right)}, b_1 = 0.$$

Substituting Set 1-Set 7 into Eq. (3.4) along with Eq. (2.7)-Eq. (2.10), we get the following families of traveling wave solutions:

Hyperbolic function solutions: When $\mu < 0$, we get the following seven families of hyperbolic function solutions.

Family 1: $u_1(\xi) = \mp\frac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{-\mu}\left\{\tanh\left(A + \sqrt{-\mu}\xi\right) + \coth\left(A + \sqrt{-\mu}\xi\right)\right\}\right),$

where $\xi = x - \left(\frac{q^2 - 48r^2\mu}{6r}\right)t$.

Family 2: $u_2(\xi) = \pm\frac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{-\mu}\tanh\left(A + \sqrt{-\mu}\xi\right)\right),$

$$u_3(\xi) = \pm\frac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{-\mu}\coth\left(A + \sqrt{-\mu}\xi\right)\right),$$

where $\xi = x - \left(\frac{q^2 - 12r^2\mu}{6r}\right)t$.

Family 3: $u_4(\xi) = \pm\frac{1}{\sqrt{6pr}}\left(q + 3r\sqrt{-\mu}\left\{\tanh\left(A + \sqrt{-\mu}\xi\right) \mp I\sec h\left(A + \sqrt{-\mu}\xi\right)\right\}\right),$

$$u_5(\xi) = \pm\frac{1}{\sqrt{6pr}}\left(q + 3r\sqrt{-\mu}\left\{\coth\left(A + \sqrt{-\mu}\xi\right) \mp \csc h\left(A + \sqrt{-\mu}\xi\right)\right\}\right),$$

where $\xi = x - \left(\frac{q^2 - 3r^2\mu}{6r}\right)t$.

Family 4: $u_6(\xi) = \mp\frac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{-\mu}\coth\left(A + \sqrt{-\mu}\xi\right)\right),$

$$u_7(\xi) = \mp\frac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{-\mu}\tanh\left(A + \sqrt{-\mu}\xi\right)\right),$$

where $\xi = x - \left(\frac{q^2 - 12r^2\mu}{6r}\right)t$.

Family5: $u_8(\xi) = \mp\frac{1}{\sqrt{6pr}}\begin{pmatrix}(6r\mu\lambda - q) - 6r\left(\mu\lambda^2 + 1\right)\sqrt{-\mu} \\ \times\left(\coth\left(A + \sqrt{-\mu}\xi\right) + \lambda\sqrt{-\mu}\right)^{-1}\end{pmatrix},$

$$u_9(\xi) = \mp\frac{1}{\sqrt{6pr}}\begin{pmatrix}(6r\mu\lambda - q) - 6r\left(\mu\lambda^2 + 1\right)\sqrt{-\mu} \\ \times\left(\tanh\left(A + \sqrt{-\mu}\xi\right) + \lambda\sqrt{-\mu}\right)^{-1}\end{pmatrix},$$

where $\xi = x - \left(\frac{q^2 - 12r^2\mu}{6r}\right)t$.

Family 6: $u_{10}(\xi) = \mp\frac{1}{\sqrt{6pr}}\left((3r\mu\lambda + q) + 3r\sqrt{-\mu}\begin{Bmatrix}\coth\left(A + \sqrt{-\mu}\xi\right) \\ \pm\operatorname{csc}h\left(A + \sqrt{-\mu}\xi\right) + \lambda\sqrt{-\mu}\end{Bmatrix}\right),$

$$u_{11}(\xi) = \mp\frac{1}{\sqrt{6pr}}\left((3r\mu\lambda + q) + 3r\sqrt{-\mu}\begin{Bmatrix}\tanh\left(A + \sqrt{-\mu}\xi\right) \\ \pm I\operatorname{sec}h\left(A + \sqrt{-\mu}\xi\right) + \lambda\sqrt{-\mu}\end{Bmatrix}\right),$$

where $\xi = x - \left(\frac{q^2 - 3r^2\mu}{6r}\right)t$.

Family 7: $u_{12}(\xi) = \mp\frac{1}{2}\sqrt{\frac{6r}{p}}\sqrt{-\mu}\begin{Bmatrix}\coth\left(A + \sqrt{-\mu}\xi\right) \\ \pm\operatorname{csc}h\left(A + \sqrt{-\mu}\xi\right) - \frac{q}{3r\mu}\sqrt{-\mu}\end{Bmatrix},$

$$u_{13}(\xi) = \mp\frac{1}{2}\sqrt{\frac{6r}{p}}\sqrt{-\mu}\left\{\begin{array}{l}\tanh\left(A+\sqrt{-\mu}\xi\right) \\ \pm I\sec h\left(A+\sqrt{-\mu}\xi\right)-\dfrac{q}{3r\mu}\sqrt{-\mu}\end{array}\right\},$$

where $\xi = x - \left(\dfrac{q^2 - 3r^2\mu}{6r}\right)t$.

Trigonometric function solutions: When $\mu > 0$, we get the following seven families of trigonometric function solutions.

Family :8 $u_{14}(\xi) = \mp\dfrac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{\mu}\left\{\tan\left(A-\sqrt{\mu}\xi\right) - \cot\left(A-\sqrt{\mu}\xi\right)\right\}\right)$,

$$u_{15}(\xi) = \mp\frac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{\mu}\left\{\cot\left(A+\sqrt{\mu}\xi\right) - \tan\left(A+\sqrt{\mu}\xi\right)\right\}\right)$$

where $\xi = x - \left(\dfrac{q^2 - 48r^2\mu}{6r}\right)t$.

Family 9: $u_{16}(\xi) = \pm\dfrac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{\mu}\tan\left(A-\sqrt{\mu}\xi\right)\right)$,

$$u_{17}(\xi) = \pm\frac{1}{\sqrt{6pr}}\left(q + 6r\sqrt{\mu}\cot\left(A+\sqrt{\mu}\xi\right)\right),$$

where $\xi = x - \left(\dfrac{q^2 - 12r^2\mu}{6r}\right)t$.

Family 10: $u_{18}(\xi) = \pm\dfrac{1}{\sqrt{6pr}}\left(q + 3r\sqrt{\mu}\left\{\tan\left(A-\sqrt{\mu}\xi\right) \mp \sec\left(A-\sqrt{\mu}\xi\right)\right\}\right)$,

$$u_{19}(\xi) = \pm\frac{1}{\sqrt{6pr}}\left(q + 3r\sqrt{\mu}\left\{\cot\left(A+\sqrt{\mu}\xi\right) \mp \csc\left(A+\sqrt{\mu}\xi\right)\right\}\right),$$

where $\xi = x - \left(\dfrac{q^2 - 3r^2\mu}{6r}\right)t$.

Family 11: $u_{20}(\xi) = \mp \frac{1}{\sqrt{6pr}} \left(q - 6r\sqrt{\mu} \cot\left(A - \sqrt{\mu}\xi \right) \right),$

$$u_{21}(\xi) = \mp \frac{1}{\sqrt{6pr}} \left(q - 6r\sqrt{\mu} \tan\left(A + \sqrt{\mu}\xi \right) \right),$$

where $\xi = x - \left(\frac{q^2 - 12r^2\mu}{6r} \right) t$.

Family12: $u_{22}(\xi) = \mp \frac{1}{\sqrt{6pr}} \begin{pmatrix} (6r\mu\lambda - q) - 6r(\mu\lambda^2 + 1)\sqrt{\mu} \\ \times \left(\cot\left(A - \sqrt{\mu}\xi \right) + \lambda\sqrt{\mu} \right)^{-1} \end{pmatrix},$

$$u_{23}(\xi) = \mp \frac{1}{\sqrt{6pr}} \begin{pmatrix} (6r\mu\lambda - q) - 6r(\mu\lambda^2 + 1)\sqrt{\mu} \\ \times \left(\tan\left(A + \sqrt{\mu}\xi \right) + \lambda\sqrt{\mu} \right)^{-1} \end{pmatrix},$$

where $\xi = x - \left(\frac{q^2 - 12r^2\mu}{6r} \right) t$.

Family13: $u_{24}(\xi) = \mp \frac{1}{\sqrt{6pr}} \left((3r\mu\lambda + q) - 3r\sqrt{\mu} \begin{Bmatrix} \cot\left(A - \sqrt{\mu}\xi \right) \\ \pm \csc\left(A - \sqrt{\mu}\xi \right) + \lambda\sqrt{\mu} \end{Bmatrix} \right),$

$$u_{25}(\xi) = \mp \frac{1}{\sqrt{6pr}} \left((3r\mu\lambda + q) - 3r\sqrt{\mu} \begin{Bmatrix} \tan\left(A + \sqrt{\mu}\xi \right) \\ \pm \sec\left(A + \sqrt{\mu}\xi \right) + \lambda\sqrt{\mu} \end{Bmatrix} \right),$$

where $\xi = x - \left(\frac{q^2 - 3r^2\mu}{6r} \right) t$.

Family 14: $u_{26}(\xi) = \mp \frac{1}{2} \sqrt{\frac{6r}{p}} \sqrt{\mu} \left\{ \cot\left(A - \sqrt{\mu}\xi \right) \pm \csc\left(A - \sqrt{\mu}\xi \right) - \frac{q}{3r\mu}\sqrt{\mu} \right\},$

$$u_{27}(\xi) = \mp \frac{1}{2} \sqrt{\frac{6r}{p}} \sqrt{\mu} \left\{ \tan\left(A + \sqrt{\mu}\xi \right) \pm \sec\left(A + \sqrt{\mu}\xi \right) - \frac{q}{3r\mu}\sqrt{\mu} \right\},$$

where $\xi = x - \left(\dfrac{q^2 - 3r^2\mu}{6r} \right) t$.

Remark: All the obtained solutions have been checked with Maple by putting them back into the original equations and found correct.

4. Discussion

- If we set $p = q = r = 1$, $A = 0, \mu = -1$ into $u_2(\xi)$ within the interval $-3 \le x, t \le 3$, we get a kink wave, which is represented in Fig. 1.

- Fig. 2 is a soliton wave profile of $u_4(\xi)$ for $p = q = r = 1$, $A = -7, \mu = -1$ within the interval $-3 \le x, t \le 3$.

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- For $p = 4, q = 1, r = 5$, $A = 0, \mu = 8$ within the interval $-3 \le x, t \le 3$ and $p = q = 1, r = 8$, $A = 0, \mu = 1$ within the interval $-2 \le x, t \le 2$, $u_{16}(\xi)$ and $u_{18}(\xi)$ show the shape of periodic traveling wave solutions represented in Fig. 3 and Fig. 4 respectively.

Some of our obtained traveling wave solutions are represented in Figs (1- 4).

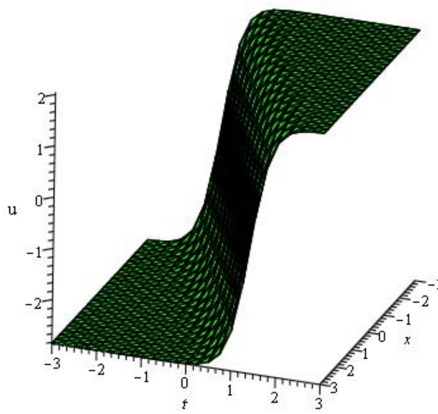

Fig. 1. Kink shape of $u_2(\xi)$ for $p = q = r = 1$, $A = 0, \mu = -1$ within the interval $-3 \le x, t \le 3$.

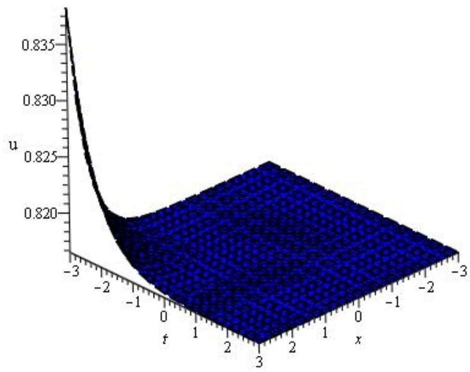

Fig. 2. Soliton wave profile of $u_4(\xi)$ for $p = q = r = 1$, $A = -7, \mu = -1$ within the interval $-3 \le x, t \le 3$.

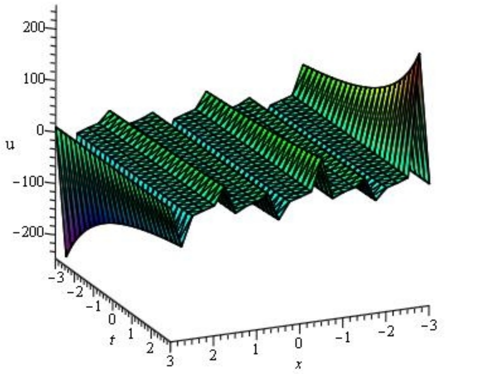

Fig. 3. Periodic wave profile of $u_{16}(\xi)$ for $p=4,\ q=1, r=5$, $A=0, \mu=8$ within the interval $-3 \le x, t \le 3$.

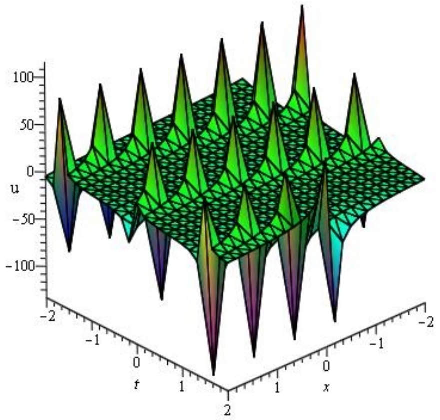

Fig. 4. Periodic wave profile of $u_{18}(\xi)$ for $p=q=1,\ r=8$, $A=0, \mu=1$ within the interval $-2 \le x, t \le 2$.

5. Conclusions

In this paper, we focus our attention on the enhanced (G'/G)-expansion method to derive exact traveling wave solutions of modified Burgers-KDV equation. An abundant set of solutions of a variety of distinct physical structures such as solitons, singular solitons and periodic solutions were formally derived. These solutions have both singular and nonsingular behaviors depending upon the choice of the parameters. Finally, it is worth mentioning that the enhanced (G'/G)-expansion method is reliable and effective and gives more solutions. This method can also be applied to other kinds of nonlinear partial differential equations. The availability of computer systems like Maple facilitates the tedious algebraic calculations.

Competing Interests

Authors have declared that no competing Interests exist.

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