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A Fuzzy Approach for Solving Three–Level Chance Constrained Quadratic Programming Problem

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Abstract

This paper presents a fuzzy approach for solving a three-decision maker's model and presents how to solve three–level chance constraints quadratic programming problem. After converting probabilistic nature of the constraints to equivalence deterministic constraints each level attempts to optimize its problem separately using fuzzy programming technique, in this method the tolerance and membership function concepts are used to develop Tchebycheff problem for generating Pareto optimal solution for this problem. Finally, a numerical example is given to clarify the main results developed in this paper.

Keywords: Three–level; chance constraints; fuzzy programming.

1 Introduction

Multi–level optimization plays an important role in engineering design, management and decision making in general. Ultimately, a designer or decision maker needs to make tradeoffs between disparate and conflicting design objectives. The field of three–level optimization defines the art and science of making such decisions. The prevailing approach to address this decision-making task is to solve an optimization problem, which yields a candidate solution [1,2,3].

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Multilevel programming techniques are developed to solve decentralized planning problems with multiple decision makers in a hierarchal organization [4,5].

Three-level programming is a class of multi–level programming in which there are three independent decision-makers (DMs). Each DM attempts to optimize its objective function and is affected by the actions of the others DMs [2].

The basic concept of the fuzzy programming approaches implies that the LLDM optimizes his/her objective function, taking a goal or preference of the ULDM into consideration. In the decision process, considering the membership functions of the fuzzy goals for the decision variables of the ULDM, the LLDM solves a FP problem with the set of constraints on an overall satisfactory degree of the ULDM. If the proposed solution is not satisfactory to the ULDM, the solution search is continued by redefining the elicited membership functions until a satisfactory solution is reached [6,7].

Fuzzy approach uses the concept of tolerance membership to develop a fuzzy max-min decision model for generating Pareto optimal (satisfactory) solution for three level programming problem; the first level decision maker (FLDM) specifies his objective functions and decisions with possible tolerances which are described by membership functions of fuzzy set theory. Then, the second level decision maker (SLDM) specifies his objective functions and decisions, in the view of FLDM, with possible tolerances which are described by membership functions of fuzzy set theory. Finally, the third level decision maker (TLDM) uses the preference information for upper levels decision – maker subject to the upper levels decision [2,3,8].

Decision problems of chance constrained or stochastic optimization arises when certain coefficient of an optimization model is not fixed or known, but are instead, to some extent, probabilistic quantities. In most of the real life problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extreme problems under incomplete information about the random parameters is called stochastic programming [9,10].

In [11] Pramanik and Banerjee dealt with a fuzzy goal programming approach to solve chance constrained quadratic bi-level programming problem. Chance constraints were converted into equivalent deterministic constraints by the prescribed distribution functions. In this model formulation, the quadratic membership functions are formulated by using the individual best solution of the quadratic objective functions subject to the equivalent deterministic constraint.

Therefore Pramanik et al. [12] presented a fuzzy goal programming approach to solve chance constrained linear plus linear fractional bi-level programming problem. The chance constraints with right hand parameters as random variables of prescribed probability distribution functions are transformed into equivalent deterministic system constraints. They construct nonlinear membership functions based on deterministic system constraints. The nonlinear membership functions are transformed into linear membership functions by using first order Taylor's series approximation. In the bi-level decision making context. In this paper some simple and easy techniques are used to solve the problem like the technique which used in [13] to convert the probabilistic nature of the constraints to equivalence deterministic constraints , simplex method to give the best and the worst individual solution for each decision maker's problem, the fuzzy set theory to formulate the previous results to membership function and use the concept of tolerance to develop Tchebycheff problem for generating Pareto optimal solution for our problem

In [14] Sadand Emam presented a solution algorithm to solve bi-level integer linear fractional programming problem with individual chance constraints (CHBLIFP). They assumed that there is randomness in the right-hand side of the constraints only and that the random variables are normally distributed. The basic idea in treating (CHBLIFP) they dealt with is to convert the probabilistic nature of this problem into a deterministic bi-level integer linear fractional programming problem (BLIFP).

This paper is divided into the follow sections: section 2 presents the problem formulation and solution concept of the model of the three–level chance constraints quadratic programming problem, section 3 introduce how to convert the probabilistic nature of the constraints to equivalence deterministic constraints, section 4 uses Fuzzy approach to solve the multilevel chance constraints quadratic programming problem, in section 5 a numerical example is provided to clarify the results. Finally, concluding remarks and future works are given in section 6.

2 Problem Formulation and Solution Concept

Let $x_j \in R^n, (j = 1,2,3)$ be a vector variable indicating the first decision level choice, the second decision level choice, and the third decision level choice, $n = \sum_{j=1}^{3} n_j$.

Let $F_i : R^n \to R^{N_i}, (i = 1,2,3)$ be the first level objective function, the second level objective function, and the third level objective function respectively. Let the FLDM (First Level Decision Maker), SLDM (Second Level Decision Maker), and TLDM (Third Level Decision Maker) have N_1, N_2 and N_3 objective function, respectively.

Therefore, the three–level chance constraints quadratic programming (TLCCQP) problem may be formulated as follows:

[First Level]

$$\max_{x_1} f_1(x) = kx + \tfrac{1}{2}x^T Cx, \tag{1}$$

Where x_2, x_3 solves

[Second Level]

$$\max_{x_2} f_2(x) = kx + \tfrac{1}{2}x^T Cx, \tag{2}$$

Where x_3 solves

[Third Level]

$$\max_{x_3} f_3(x) = kx + \tfrac{1}{2}x^T Cx, \tag{3}$$

Subject to

$$(x_1, x_2, x_3) \in G. \tag{4}$$

Where

$$G = \{ x \in R^n : pr(g_i(x) = \sum_{j=1}^{n} c_{ij} x_j \le v_i) \ge \alpha_i ,\ (i = 1,2,...,n), x_1, x_2, x_3 \ge 0 \}. \tag{5}$$

Where f_1, f_2 and $f_3$are the objective functions of the first level decision maker (FLDM), second level decision maker (SLDM), and third level decision maker (TLDM), C is $n \times n$ real matrix, and k is $(1 \times n)$ matrix, the vector of decision variables x is n-vector partitioned between the three planners.

Definition 1.

For any $\left(x_1 \in G_1 = \{x_1 | (x_1, x_2, x_3) \in G\}\right)$ given by FLDM and $\left(x_2 \in G_2 = \{x_2 \ | (x_1, x_2, x_3) \in G\}\right)$ given by SLDM, if the decision-making variable $\left(x_3 \in G_3 = \{x_3 \ | \ (x_1, x_2, x_3) \in G\}\right)$ is the optimal solution of the TLDM, then (x_1, x_2, x_3) is the feasible solution of TLCCQP problem.

Definition 2.

If (x_1^*, x_2^*, x_3^*) is a feasible solution of the TLCCQP problem; no other feasible solution $(x_1, x_2, x_3) \in G$ exists; so $(x_1^*, x_2^*, x_3^*)$is the optimal solution of the TLCCQP problem.

3 Converting Probabilistic Nature of the Constraints to Equivalence Deterministic Constraints

The basic idea in this section is to convert the probabilistic nature of the constraints to equivalence deterministic constraints. Using:

$$\overline{G} = \{ \ x \in R^n \Big| \ \sum_{j=1}^{n} c_{ij} x_j \le E(b_i) + K_{\alpha_i} \sqrt{Var(b_i)}, i = 1,2,...n, x_j \ge 0.\} \text{ [13]} \quad (6)$$

Assume that $\overline{G}$ is the constraints after converting from probabilistic nature to equivalence deterministic constraints, the random parameters b_i ,(i = 1, 2… n), $E(b_i)$ is the mean, $Var(b_i)$ is the variance and K_{α_i} is the standard normal table value of the constraint.

So the TLCCQP problem can be written as:

[First Level]

$$\max_{x_1} f_1(x) = kx + \frac{1}{2} x^T C x, \quad (7)$$

Where x_2, x_3 solves

[Second Level]

$$\max_{x_2} f_2(x) = kx + \frac{1}{2} x^T C x, \quad (8)$$

Where x_3 solves

[Third Level]

$$\max_{x_3} f_3(x) = kx + \frac{1}{2} x^T C x, \quad (9)$$

Subject to

$$(x_1, x_2, x_3) \in \overline{G}. \tag{10}$$

Where

$$\overline{G} = \{ x \in R^n : g_i(x) = \sum_{j=1}^{n} c_{ij} x_j \le d,\ (i = 1,2,...,n),\ x_1, x_2, x_3 \ge 0 \} \tag{11}$$

Where d = $E(b_i) + K_{\alpha_i} \sqrt{Var(b_i)}$.

4 Fuzzy Approach of the Three–Level Chance Constraints Quadratic Programming Problem

To solve the three–level chance constraints quadratic programming problem, one first convert the probabilistic nature of the constraints to equivalence deterministic constraints, after that gets the satisfactory solution that is acceptable to FLDM, and then gives the FLDM decision variable and goal with some leeway to the SLDM for him/her to seek the optimal solution, then the SLDM give the decision variables and goals with some leeway to the TLDM for him/her to seek the satisfactory solution and to arrive at the solution which is closer to the optimal solution of the FLDM. This due to, the TLDM who should not only optimize his/her objective function, but also try to satisfy the SLDM goals and preferences as much as possible, SLDM also do the same action to satisfy the FLDM goals and preferences as much as possible.

4.1 FLDM Problem

First, the FLDM solves (7) subject to $\overline{G}$, by applying the simplex method. The individual best solution (F_1^*) and individual worst solution (F_1^-) of (7) subject to $\overline{G}$ are:

$$F_1^* = \underset{x \in \overline{G}}{Max} F_1(x) \quad , \ F_1^-(x) = \underset{x \in \overline{G}}{Min} F_1(x) \tag{12}$$

Goals and tolerances can then be reasonably set for individual solution and the differences of the best and worst solutions, respectively. This data can then be formulated as the following membership function of fuzzy set theory [8]:

$$\mu\ [F_1(x)] = \begin{cases} 1 & if \quad F_1(x) > F_1^*, \\ \dfrac{F_1(x) - F_1^-}{F_1^* - F_1^-} & if \quad F_1^- \le F_1(x) \le F_1^*, \\ 0 & if \quad F_1^- \ge F_1(x). \end{cases} \tag{13}$$

4.2 SLDM Problem

In the same way, the SLDM independently solves (8) subject to $\overline{G}$, by applying the simplex method. The individual best solution (F_2^*) and individual worst solution (F_1^-) of (8) subject to $\overline{G}$ are:

$$F_2^* = \underset{x \in \overline{G}}{Max}\, F_2(x) \quad , F_2^-(x) = \underset{x \in \overline{G}}{Min}\, F_2(x) \tag{14}$$

This information can then be formulated as the following membership function:

$$\mu\ [F_2(x)] = \begin{cases} 1 & if \quad F_2(x) > F_2^*, \\ \dfrac{F_2(x) - F_2^-}{F_2^* - F_2^-} & if \quad F_2^- \le F_2(x) \le F_2^*, \\ 0 & if \quad F_2^- \ge F_2(x). \end{cases} \tag{15}$$

4.3 TLDM Problem

In the same way, the TLDM independently solves (9) subject to $\overline{G}$, by applying the simplex method. The individual best solution (F_3^*) and individual worst solution (F_3^-) of (9) subject to $\overline{G}$ are:

$$F_3^* = \underset{x \in \overline{G}}{Max}\, F_3(x) \quad , F_3^-(x) = \underset{x \in \overline{G}}{Min}\, F_3(x) \tag{16}$$

This information can then be formulated as the following membership function:

$$\mu\ [F_3(x)] = \begin{cases} 1 & if \quad F_3(x) > F_3^*, \\ \dfrac{F_3(x) - F_3^-}{F_3^* - F_3^-} & if \quad F_3^- \le F_3(x) \le F_3^*, \\ 0 & if \quad F_3^- \ge F_3(x). \end{cases} \tag{17}$$

Now the solution of the FLDM, SLDM and TLDM are disclosed. However, three solutions are usually different because of nature between three levels objective functions. The FLDM and SLDM know that using the optimal decisions, x_2^S as a control factor for the TLDM are not practical. It is more reasonable to have some tolerance that gives the TLDM an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions.

In this way, the range of decision variable x_1, x_2 should be around x_1^F , x_2^S with maximum tolerance t_1, t_2 and the following membership function specify x_1^F, x_2^S as:

$$\mu_{x_1}(x_1) = \begin{cases} \dfrac{x_1 - (x_1^F - t_1)}{t_1} & x_1^F - t_1 \le x_1 \le x_1^F, \\ \dfrac{(x_1^F + t_1) - x_1}{t_1} & x_1^F \le x_1 \le x_1^F + t_1. \end{cases} \tag{18}$$

$$\mu_{x_2}(x_2)=\begin{cases}\dfrac{x_2-(x_2^S-t_2)}{t_2} & x_2^S-t_2\le x_2\le x_2^S,\\[2ex] \dfrac{(x_2^S+t_2)-x_2}{t_2} & x_2^S\le x_2\le x_2^S+t_2.\end{cases} \tag{19}$$

Finally, in order to generate the satisfactory solution, which is also the optimal solution with overall satisfaction for all decision - makers, the Tchebycheff problem can be solved as the following: [15]

$$Max\,\delta, \tag{20}$$

Subject to

$$\frac{[(x_1^F+t_1)-x_1]}{t_1}\ge \delta I,$$

$$\frac{[x_1-(x_1^F-t_1)]}{t_1}\ge \delta I,$$

$$\frac{[(x_2^F+t_2)-x_2]}{t_2}\ge \delta I,$$

$$\frac{[x_2-(x_2^F-t_2)]}{t_2}\ge \delta I,$$

$$\mu[F_1(x)]\ \ge\ \delta,$$

$$\mu[F_2(x)]\ \ge\ \delta,$$

$$\mu[F_3(x)]\ \ge\ \delta,$$

$$(x_1,x_2,x_3)\in G,$$

$$t_1,t_2\ >\ 0,$$

$$\delta\in[0,1].$$

Where δ is the overall satisfaction, and I is the column vector with all elements equal to 1s. By solving problem (20). If the FLDM is satisfied with the solution then optimal solution is reached. Otherwise, he/she should provide a new membership function for the control variable and objectives to the SLDM and TLDM, until an optimal solution is reached. It is easy to see that there is an inverse correlation between t_1,t_2 and δ .

5 Numerical Example

To demonstrate the solution for (TLLSLP) problem, let us consider the following example:

[Upper Level]

$$\underset{x_1}{Max}\ F_1(x_1) = \underset{x_1,}{Max} \quad 5x_1^2 + 3x_2^2 + x_3^2,$$

Where x_2 solves

[Second Level]

$$\underset{x_2}{Max} F_2(x_2) = \underset{x_2}{Max} \quad x_1 + 5x_2^2 + x_3^2,$$

Where x_3 solves

[Third Level]

$$\underset{x_3}{Max}\, F_3(x_3) = \underset{x_3}{Max} \quad x_1^2 + x_2 + 4x_3^2,$$

Subject to

$$(x_1, x_2, x_3) \in G.$$

Where G=

$$\begin{aligned} \{(x_1, x_2, x_3) | \, & pr(x_1 + x_2 + x_3 \le b_1) \ge 0.7, \\ & pr(x_1 + 2x_2 + x_3 \le b_2) \ge 0.8, \\ & pr(x_1 + x_2 + 2x_3 \le b_3) \ge 0.93, \\ & x_1, x_2, x_3 \ge 0. \ \}. \end{aligned}$$

Suppose that$b_{i,}$ (i=1,2,3) is normally distributed random parameters with the following means and variances, E(b1)=10, E(b2)=14, E(b2)=12,VAR(b1)=16,VAR(b2)=25,VAR(b3)=4 .

From the standers normal table:$k_{\alpha1}$=0.758, $k_{\alpha2}$=0.7881,$k_{\alpha3}$=0.8238.

1. In the first the probability nature will be converted to equivalent deterministic constrain.
2. Secondly, reformulate the problem in to deterministic form as following:

[Upper Level]

$$\underset{x_1}{Max}\ F_1(x_1) = \underset{x_1,}{Max} \quad 5x_1^2 + 3x_2^2 + x_3^2,$$

Where x_2 solves

[Second Level]

$$\underset{x_2}{Max} F_2(x_2) = \underset{x_2}{Max} \qquad x_1 + 5x_2^2 + x_3^2$$

Where x_3 solves

[Third Level]

$$\underset{x_3}{Max} F_3(x_3) = \underset{x_3}{Max} \qquad x_1^2 + x_2 + 4x_3^2,$$

Subject to

$$(x_1, x_2, x_3) \in \overline{G},$$
$$x_1 + x_2 + x_3 \leq 13.03,$$
$$x_1 + 2x_2 + x_3 \leq 17.940,$$
$$x_1 + x_2 + 2x_3 \leq 13.647,$$
$$x_1, x_2, x_3 \geq 0.$$

First, the FLDM solves his/her problem as follows:

Find individual best and worst solutions by solving the FLDM problem using simplex methodas $F_1^* = 849$, and $F_1^-(x) = 0$.

Second, the SLDM solves his/her problem as follows:
Find individual best and worst solutions by solving the SLDM problem using simplex method as $F_2^* = 402.30$, and $F_2^-(x) = 0$.

Third, the TLDM solves his/her problem as follows:

Find individual best and worst solutions by solving the TLDM problem using simplex method $F_3^* = 186.04$, and $F_3^-(x) = 0$.

Finally

1. Assume the FLDM control decision x_1^F is around (0) with tolerance1.
2. Assume the SLDM control decision x_2^F is around (8.97) with the tolerance 2.

Max δ,

Subject to

$$(x_1, x_2, x_3) \in \overline{G},$$

$$5x_1^2 + 3x_2^2 + x_3^2 - 849\delta \geq 0,$$

$$x_1 + 5x_2^2 + x_3^2 - 402.30\delta \geq 0,$$

$$x_1^2 + x_2 + 4x_3^2 - 186.04\delta \geq 0,$$

$$x_1 + \delta \leq 14.03,$$

$$x_1 - \delta \geq 12.03,$$

$$x_2 + 2\delta \leq 10.970,$$

$$x_2 - 2\delta \geq 6.97,$$

$$\delta \in [0,1].$$

Whose compromise solution is $(x_1, x_2, x_3) = (13.02, 8.26, 3)$ and $\delta = 0.67$ (overall satisfaction for all decision maker's) and the objective function value are F1= 927.829, F2=90.247,F3= 213.78.

6 Summary and Concluding Remarks

In this paper three-level chance constrained quadratic programming problem (TLCCQP) is presented by using fuzzy approach which is easy to apply and simple to be understood. After converting probabilistic nature of the constraints to equivalence deterministic constraints the FLDM solved his/her problem and give the individual best solution and individual worst solution by applying the simplex method, the SLDM solved his/her problem and give the individual best solution and individual worst solution by applying the simplex method, TLDM solved his/her problem and give the individual best solution and individual worst solution by applying the simplex method, after that the membership functions were created using the tolerance concept to develop Tchebycheff problem for generating Pareto optimal solution for this problem.

However, there are many other aspects, which should by explored and studied in the area of stochastic multi-level optimization such as:

1- A three–level integer programming problem with stochastic parameters in objective function.
2- A three–level chance constraints quadratic mixed integer programming problem with stochastic parameters in objective function.

Competing Interests

Authors have declared that no competing interests exist.

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