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A Solution of Infiltration Problem Arising in Farmland Drainage Using Adomian Decomposition Method

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Authors' contributions

This work was carried out in collaboration between all authors. Author TP designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript and managed literature searches. Authors RM managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In this paper, A farmland drainage problem considered and Adomian decomposition method has been discussed and applied to solve the one-dimensional unstable flow equation that is under uniform intensity of infiltration. Convergence analysis of the Adomian decomposition method has been discussed to test the convergence of the method. Finally Maple-16 software has been used and run on a system having core i7 and 64 bit processor to obtain the Physical interpretation and the numerical values of the given problem.

Keywords: Farmland drainage; infiltration; adomian decomposition method; convergence analysis.

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1. INTRODUCTION

"Agricultural drainage systems" are systems that make it easier for water to flow from the land, so that agriculture can benefit from the subsequently reduced water levels. The systems can be made to ease the flow of water over the soil surface or through the underground, which leads to a distinction between "surface drainage systems" and "subsurface drainage systems". Both types of systems need an internal or "field drainage system", which lowers the water level in the field and an external or "main drainage system", which transports the water to the outlet.

The movement of water in a natural soil starts from a soil surface and ends at a groundwater reservoir. The flow process of water from soil surface into the soil is called infiltration. Infiltration depends on sufficient porosity in the surface soil for rainfall to infiltrate, and in the subsoil and parent material (if shallow) for rainwater to percolate. When the porosity of the surface soil is too low to accept rainfall, or subsoil porosity is too low to allow rainwater percolation (i.e. permeability is too slow), then infiltration will be restricted and rainwater will be lost as runoff. Ground water level rises up when infiltration amount over displacement and its level goes down when infiltration amount less than displacement and it keeps invariant when infiltration amount is equal to displacement. The farmland drainage under infiltration is an unstable process [1,2,3] and infiltration has great influence on ground water [4]. Groundwater level fluctuates over time. It depends on the intensity of infiltration. Technically speaking, the level converts unstable state from stable state. Thus, Ground water status under infiltration is relatively complicated. In agriculture production, we always seek how groundwater level will be under the influence of a given infiltration amount whether it rises up or falls down?

Generally, mathematical description of above phenomena can be attributed to nonlinear partial differential equation [5] and two approaches are adopted to solve the problem in practical application: Analytical method and Numerical method.

The Adomian Decomposition method (ADM) [6] is a well-known semi analytical method for practical solution of linear or nonlinear and deterministic or stochastic operator equations, including ordinary differential equations (ODEs), partial differential equations (PDEs), integral equations, integro-differential equations, etc. The ADM is a powerful technique, which provides efficient algorithms for analytic approximate solutions and numeric simulations for real-world applications in the applied sciences and engineering. The method provides the solution in a rapid convergent series with computable terms.

Here in Section 2 the mathematical formulation of the problem has been discussed, and then section 3 and 4 discusses the analysis and convergence of the Adomian Decomposition method respectively. Section 5 discusses the Solution of the problems by using Adomian Decomposition Method. Finally the interpretation of the results and graphs are concluded in section 6.

2. MATHEMATICAL MODEL

When the infiltration at a uniform intensity, the Boussinesq equation of one-dimensional drainage problem is given by,

$$\frac{\partial h}{\partial t}-\frac{k}{\mu}\left[\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)\right]=\frac{\omega}{\mu} \tag{1}$$

with the initial condition

$$h(x,0)=h(x) \qquad 0\le x\le L \tag{2}$$

and boundary conditions

$$\begin{aligned} h(0,t)&=h_0 \qquad t\ge 0\\ h(L,t)&=h_0 \qquad t\ge 0 \end{aligned} \tag{3}$$

Where h is groundwater level with x-axis as the base (m), k is permeability coefficient (m/d), μ is soil water specific yield, ω is intensity of uniform infiltration (m/d), and t is time of drainage. $h(x)$ is a defined function that describes the water table height initially.

Dimensionless variable for the given problem are chosen as follows

$$H=\frac{h}{L},\ X=\frac{x}{L},\ T=\frac{kt}{\mu L} \text{ and } B=\frac{\omega}{k} \tag{4}$$

To convert the nonlinear governing Eq. (1) and its initial condition, we transformed into its non-dimensional form as

$$\frac{\partial H}{\partial T}-\frac{\partial}{\partial X}\left(H\frac{\partial H}{\partial X}\right)=B \quad (5)$$

along with the non-dimensional initial conditions is,

$$H(X,0)=H(X); \quad 0\le X\le 1 \quad (6)$$

The physical meaning of the linear model is that when infiltrating uniformly, both ditch-water level and river level maintains the same position with h_0. Groundwater flow towards drainage ditch when infiltrating and groundwater level is influenced by drainage ditch and infiltration. In order to determine whether groundwater level fluctuates over time, we just need to study the fluctuation change of arbitrary point on groundwater level.

3. ANALYSIS OF THE ADOMIAN METHODS

In the early 1980s, a new semi analytic method was developed by George Adomian [6] in order to solve non-linear functional equations of the form

$$LH+RH+NH=g \quad (7)$$

In Eq. (7), L is the linear operator to be inverted, which usually is just the highest order differential operator, N represents the non-linear part, R represents the remainder operator of the linear operator or lower order terms, and g is the non-homogeneous term. The solution H and the non-linearity N are assumed to have the following analytic expansions, respectively

$$H=\sum_{n=0}^{\infty}\lambda^n H_n \text{ and } NH=\sum_{n=0}^{\infty}\lambda^n A_n \quad (8)$$

Where the A_n's are the Adomian polynomials that depend only on $H_0, H_1, H_2, ...H_n$ and are given by the following formula:

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty}\lambda^k H_k\right)\right]_{\lambda=0}, n\ge 0 \quad (9)$$

In order to better explain the method, we will first assume the convergence of the series in Eq. (8) and deal with the rigorous convergence issues later. The parameter k is a dummy variable introduced for ease of computation. There are several different versions of Eq. (9) that can be found in the literature that leads to easier computation of the A_n's. It should be noted that the A_n's are the terms of analytic expansion of NH, where $H=\sum_{n=0}^{\infty}H_n\lambda^n$

In [6] Adomian has shown that the expansion for NH in Eq. (8) is a rearrangement of the Taylor series expansion of NH about the initial function H_0 in a suitable Hilbert or Banach space. Substitution of Eq. (8) in Eq. (7) results in the following:

$$L\left(\sum_{n=0}^{\infty}\lambda^n H_n\right)=-R\left(\sum_{n=0}^{\infty}\lambda^n H_n\right)-\sum_{n=0}^{\infty}\lambda^n A_n+g \quad (10)$$

The above equation can be rewritten in a recursive fashion, yielding iterates H_n, the sum of which converges to the solution H satisfying Eq. (10) if it exists

$$\sum_{n=0}^{\infty}\lambda^n H_n=-L^{-1}R\left(\sum_{n=0}^{\infty}\lambda^n H_n\right)-L^{-1}\sum_{n=0}^{\infty}\lambda^n A_n+L^{-1}(g) \quad (11)$$

On comparing the coefficient of λ on both sides of Eq. (11), It yields

$$H_0=L^{-1}(g)+H(X,0),$$
$$H_{n+1}=-L^{-1}R\left(H_n\right)-L^{-1}(A_n), \qquad n\ge 0.$$

Typically, the symbol L^{-1} represents a formal inverse of the linear operator L. In the case of partial differential equations, L is the highest order partial derivative operator for which the formal inverse can be computed using integrations. Typically convergence rates and convergence region depends upon the operator equation.

4. CONVERGENCE ANALYSIS OF THE ADOMIAN DECOMPOSITION METHODS

The first proof of convergence of the ADM was given by Cherruault [7,8], who used fixed point theorems for abstract functional equations. Since then, many articles on convergence of the ADM were published, including the works of Abbaoui and Cherruault [9,10] and Rach [11]. Furthermore, Abdelrazec and Pelinovsky [12] introduced the Convergence of the Adomian Decomposition Method for Initial-Value Problems and Duana, Rach and Wang [13] discussed the effective region of convergence of the Decomposition series solution.

We recall the following theorem from Mavoungou [14] which guarantees the convergence of Adomian's method for the general operator equation given by

$$LH + RH + NH = g$$

Consider the Hilbert space $\mathrm{H} = L^2\left(\left(\alpha,\beta\right)\times\left[0,T\right]\right)$ defined by the set of applications:

$$H:\left(\alpha,\beta\right)\times\left[0,T\right]\rightarrow R \text{ With } \int_{(\alpha,\beta)\times[0,T]} H^2\left(\eta,\xi\right)d\eta d\xi < +\infty \tag{12}$$

Let us denote

$$\begin{aligned} &LH = \frac{\partial H}{\partial T}, \quad NH = \frac{\partial}{\partial X}\left(H\frac{\partial H}{\partial X}\right), \quad RH = 0 \\ &TH = -NH - RH = -\frac{\partial}{\partial X}\left(H\frac{\partial H}{\partial X}\right) \end{aligned} \tag{13}$$

Theorem 1: Let $TH = -RH - NH$ be a hemi continuous operator in a Hilbert Space H and satisfy the following,

$$\textbf{Hypothesis}\left(\mathrm{H}_1\right):\left(TH_1 - TH_2, H_1 - H_2\right) \geq k\left\|H_1 - H_2\right\|^2, k > 0, \forall H_1, H_2 \in \mathrm{H} \tag{14}$$

Hypothesis $\left(\mathrm{H}_2\right)$**:** whatever may be $M > 0$, there exit constant $C(M) > 0$ such that for $H_1, H_2 \in \mathrm{H}$ With $\left\|H_1\right\| \leq M,\ \left\|H_2\right\| \leq M$, we have

$$\left(TH_1 - TH_2, w\right) \leq C(M)\left\|H_1 - H_2\right\|\left\|w\right\| \text{ for every } w \in \mathrm{H} \tag{15}$$

Then, for every $g \in \mathrm{H}$, the nonlinear functional equation $LH + RH + NH = g$ admit a unique solution $H \in \mathrm{H}$. Furthermore, if the solution H can be represented in a series form given by $H = \sum_{n=0}^{\infty} H_n \lambda^n$, then the Adomian decomposition scheme corresponding to the functional equation under consideration converges strongly to $H \in \mathrm{H}$, which is the unique solution to the functional equation.

Proof: Verification of hypothesis $\left(\mathrm{H}_1\right)$

$$\left(TH_1 - TH_2\right) = -\frac{1}{2}\frac{\partial^2}{\partial X^2}\left(H_1^{\,2} - H_2^{\,2}\right)$$

$$\left(TH_1 - TH_2, H_1 - H_2\right) = \left(-\frac{1}{2}\frac{\partial^2}{\partial X^2}\left(H_1^{\,2} - H_2^{\,2}\right), H_1 - H_2\right) \tag{16}$$

Since $\frac{\partial^2}{\partial X^2}$ is a differential operator in H , then there exit a real $\delta > 0$ such that,

According to Schwartz inequality, we get

$$\left(-\frac{1}{2}\frac{\partial^2}{\partial X^2}\left(H_1^{\,2}-H_2^{\,2}\right), H_1-H_2\right) \le \frac{1}{2}\delta\left\|H_1^{\,2}-H_2^{\,2}\right\|\left\|H_1-H_2\right\| \tag{17}$$

Now, we use mean value theorem, then we have

$$\begin{aligned}\left(-\frac{1}{2}\frac{\partial^2}{\partial X^2}\left(H_1^{\,2}-H_2^{\,2}\right), H_1-H_2\right) &\le \frac{1}{2}\delta\left\|H_1^{\,2}-H_2^{\,2}\right\|\left\|H_1-H_2\right\| \\ &\le \delta M\left\|H_1-H_2\right\|^2\end{aligned} \tag{18}$$

for $\|H_1\| \le M$ and $\|H_2\| \le M$.

Therefore

$$\left(-\frac{1}{2}\frac{\partial^2}{\partial X^2}\left(H_1^{\,2}-H_2^{\,2}\right), H_1-H_2\right) \ge \delta M\left\|H_1-H_2\right\|^2 \tag{19}$$

Substituting Eq. (19) in Eq. (16),

$$\left(TH_1-TH_2, H_1-H_2\right) \ge k\left\|H_1-H_2\right\|^2 \tag{20}$$

where $k = \delta M$.
Hence we find the hypothesis (H_1) .
For hypothesis (H_2) ,

$$\begin{aligned}\left(TH_1-TH_2, w\right) &= \left(\left(-\frac{1}{2}\frac{\partial^2}{\partial X^2}\left(H_1^{\,2}-H_2^{\,2}\right)\right), w\right) \\ &\le \delta M\left\|H_1-H_2\right\|\left\|w\right\| \\ &= C(M)\left\|H_1-H_2\right\|\left\|w\right\|\end{aligned} \tag{21}$$

Where $C(M) = \delta M$ and therefore hypothesis (H_2) holds.

5. SIMULATIONS RESULTS

Using the analysis of Adomian Decomposition Method, Eq. (5) can be written in operator form $L_T H$ as:

$$L_T H(X,T) = B + L_X\left(NH(X,T)\right) \tag{22}$$

Operating the inverse operators on both sides of Eq. (22), it gives

$$H(X,T) = H_0(X) + L_T^{-1}B + L_T^{-1}\left[L_X\left(NH(X,T)\right)\right] \tag{23}$$

Where $NH(X,T) = H\dfrac{\partial H}{\partial X}$ and $H_0(X)$ can be solved subject to the corresponding initial condition Eq. (6).

It is well known from Eq. (8), the solution of Eq. (5) can be written in series form as

$$H(X,T) = \sum_{n=0}^{\infty} H_n(X,T) \tag{24}$$

Where $H_0, H_1, H_2, ...H_n$ are the height of different water level at any distance X and at any time $T > 0$ and the nonlinear term can be represented as $NH(X,T) = \sum_{n=0}^{\infty} A_n$ where A_n's are the Adomian's special polynomials to be determined and defined by Eq. (9).

Following the analysis of Adomian decomposition method as discussed in [15], for the determination of the components $H_n(X,T)$ of $H(X,T)$, we set the recursive relation as

$$\sum_{n=0}^{\infty} H_n(X,T) = H_0 + L_T^{-1}\left[L_X\left(\sum_{n=0}^{\infty} A_n\right)\right] \tag{25}$$

Where $H_0 = L_T^{-1}B + H(X,0) = B*T + e^{-X}$ from Eq. (6) and

$$H_{k+1} = L_T^{-1}\left(\left(A_k\right)_X\right) \tag{26}$$

Adomian polynomials are as follows:

$$\begin{aligned}
A_0 &= H_0\left(H_0\right)_X \\
A_1 &= H_1\left(H_0\right)_X + H_0\left(H_1\right)_X \\
A_2 &= H_2\left(H_0\right)_X + H_1\left(H_1\right)_X + H_0\left(H_2\right)_X \\
A_2 &= H_3\left(H_0\right)_X + H_2\left(H_1\right)_X + H_1\left(H_2\right)_X + H_0\left(H_3\right)_X \\
&\vdots
\end{aligned} \tag{27}$$

The approximate solution in a series form up to six approximations is given by

$$\begin{aligned}
H(X,T) = {} & BT + e^{-X} + 2e^{-2X}T + \frac{1}{2}e^{-X}BT^2 + \frac{1}{8}e^{-X}B^2T^4 + \frac{10}{3}T^3e^{-2X}B \\
& + 9e^{-3X}T^2 + \frac{43}{15}T^5e^{-2X}B^2 + 30T^4e^{-3X}B + \frac{176}{3}e^{-4X}T^3 + \frac{1}{48}e^{-X}B^3T^6 \\
& + \frac{2087}{40}T^6e^{-3X}B^2 + \frac{4792}{15}T^5e^{-4X}B + \frac{59}{35}T^7e^{-2X}B^3 + \frac{2875}{6}e^{-5X}T^4 \\
& + \frac{1}{348}e^{-X}B^4T^8 + \frac{8781}{140}T^8e^{-3X}B^3 + \frac{137695}{36}T^6e^{-5X}B + \frac{57464}{63}T^7e^{-4X}B^2 \\
& + \frac{2867}{3780}T^9e^{-2X}B^4 + \frac{22932}{5}e^{-6X}T^5 + \frac{1}{3840}e^{-X}B^5T^{10} + ...
\end{aligned} \tag{28}$$

Solution of Eq. (28) has been obtained by using Maple Software in T-direction. Similarly the solution can be find out in X-direction and both the operator equations will yields series solutions which converge to the same function with different convergence regions and with different convergence rates. But here it is considered only it in T-direction for convenience of the problem. Since here the problem be dealing with dimensionless variables so the range of X and T be restricted in between 0 and 1 and region of convergence here in this problem be restricted within the rectangle $[0,1]\times[0,1]$.

6. RESULTS AND GRAPHS

For seeking a numerical verification for the results, we use primary parameters as in Table 1.

Table 1. Primary parameters

Particulars	Value
K Permeability coefficient	$1\text{x}10^{-1}$ - $1\text{x}10^{-3}$
μ Soil water specific yield	0.05
ω Intensity of uniform Infiltration	0.01
Δt Time step	0.01

Table 2. Numerical values of height (H) Vs. time (T) at B=0.5 keeping distance constant

H / T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.01	0.927	0.838	0.757	0.685	0.619	0.560	0.507	0.458	0.415	0.376
0.02	0.951	0.858	0.774	0.700	0.632	0.572	0.517	0.468	0.423	0.384
0.03	0.977	0.880	0.793	0.715	0.646	0.584	0.528	0.477	0.432	0.392
0.04	1.005	0.903	0.812	0.732	0.660	0.596	0.539	0.487	0.441	0.400
0.05	1.037	0.929	0.834	0.750	0.675	0.609	0.550	0.497	0.450	0.408
0.06	1.073	0.957	0.857	0.769	0.691	0.623	0.562	0.508	0.459	0.416
0.07	1.114	0.989	0.882	0.789	0.708	0.637	0.574	0.518	0.469	0.425
0.08	1.162	1.026	0.910	0.811	0.726	0.652	0.587	0.530	0.479	0.434
0.09	1.219	1.067	0.942	0.836	0.746	0.668	0.600	0.541	0.489	0.443
0.10	1.286	1.115	0.977	0.863	0.767	0.685	0.615	0.553	0.499	0.452

Table 3. Numerical values of height (H) vs. time (T) at B=5.0 keeping distance constant

H / T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.01	0.972	0.883	0.802	0.730	0.664	0.605	0.552	0.504	0.460	0.421
0.02	1.042	0.949	0.865	0.790	0.723	0.662	0.607	0.558	0.514	0.474
0.03	1.114	1.017	0.930	0.852	0.782	0.720	0.664	0.613	0.568	0.527
0.04	1.190	1.087	0.996	0.915	0.843	0.778	0.721	0.669	0.623	0.581
0.05	1.270	1.160	1.064	0.980	0.905	0.838	0.778	0.725	0.678	0.635
0.06	1.355	1.238	1.136	1.046	0.968	0.898	0.837	0.782	0.733	0.690
0.07	1.449	1.321	1.211	1.116	1.033	0.960	0.896	0.840	0.790	0.745
0.08	1.552	1.410	1.290	1.188	1.100	1.024	0.957	0.899	0.847	0.801
0.09	1.669	1.508	1.375	1.265	1.170	1.090	1.019	0.958	0.904	0.857
0.10	1.802	1.617	1.468	1.346	1.244	1.157	1.083	1.019	0.963	0.914

Table 4. Numerical values of height (H) vs. time (T) at B=50.0 keeping distance constant

H / T	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.01	1.424	1.335	1.254	1.181	1.116	1.056	1.003	0.955	0.911	0.872
0.02	1.951	1.857	1.773	1.697	1.629	1.567	1.512	1.463	1.418	1.377
0.03	2.487	2.387	2.298	2.218	2.147	2.083	2.025	1.974	1.927	1.886
0.04	3.036	2.927	2.831	2.746	2.670	2.602	2.542	2.488	2.440	2.396
0.05	3.603	3.482	3.376	3.283	3.201	3.129	3.064	3.007	2.956	2.910
0.06	4.199	4.058	3.937	3.833	3.742	3.663	3.593	3.531	3.477	3.428
0.07	4.839	4.666	4.523	4.401	4.298	4.208	4.131	4.063	4.003	3.951
0.08	5.547	5.324	5.145	4.997	4.874	4.770	4.681	4.604	4.538	4.480
0.09	6.363	6.059	5.822	5.633	5.480	5.354	5.249	5.160	5.083	5.018
0.10	7.344	6.911	6.584	6.331	6.132	5.972	5.842	5.734	5.644	5.567

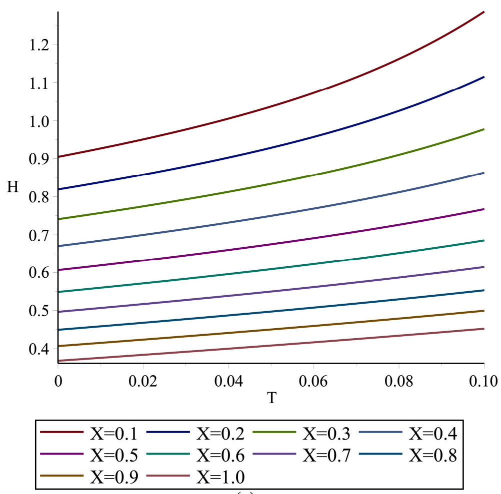

Fig. 1. Height vs. time for b=0.5

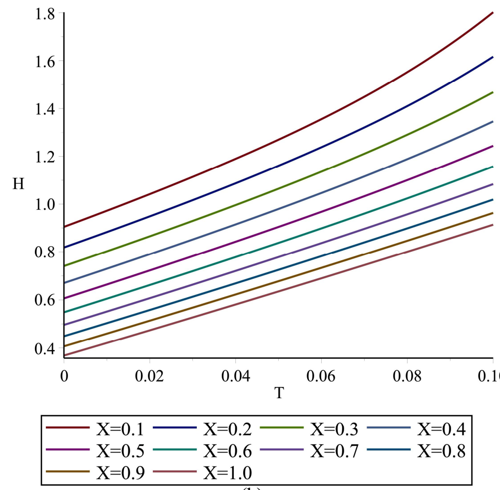

Fig. 2. Height vs. time for b=5.0

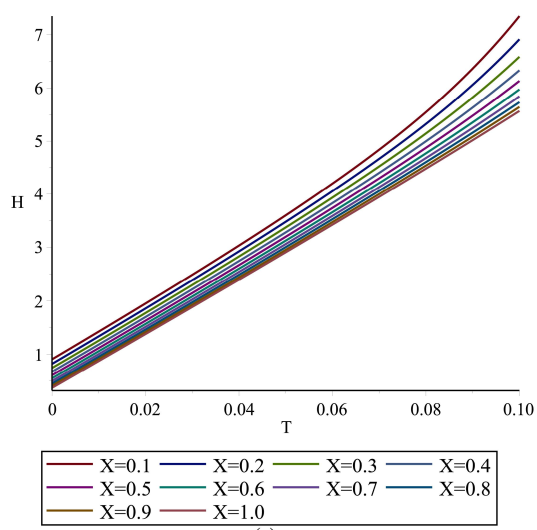

Fig. 3. Height vs. time for B=50.0

7. DISCUSSION AND CONCLUSION

Here the variation of height of water level with intensity of infiltration and time at a particular point for a farmland drainage problem has been studied. Eq. (28) represents the height of water level with the effect of intensity of infiltration effect at a particular distance that varies with time. It appears from Tables. 2, 3, 4 and graph (Figs. 1, 2 and 3) that height of water level increases for a particular intensity of infiltration with time at a fixed point, but as the intensity of infiltration increases water level increases and the height of water level increases with time due to the fact that intensity of infiltration depends upon the permeability of the medium.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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