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A Continuous Poisson-Rayleigh Distribution: Its Properties and Applications

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This article proposed a Poisson based continuous probability distribution called Poisson-Rayleigh distribution. Some properties of the new distribution such as quantile and reliability functions and other useful measures were obtained. The model parameters were estimated using the method of maximum likelihood. The usefulness of the new distribution was proven empirically using real life datasets.

Keywords: Poisson distribution; rayleigh distribution; poisson-rayleigh distribution; statistical properties, parameter estimation; applications.

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1 Introduction

The Rayleigh was obtained from the amplitude of sound resulting from many important sources by Rayleigh [1]. It is continuous probability distribution with a wide range of applications such as in life testing experiments, reliability analysis, applied statistics and clinical studies. This distribution is a special case of the two parameter Weibull distribution with the shape parameter equal to 2. Its origin and other important features can be found in the work of Siddiqui [2], Hirano [3] as well as Howlader and Hossian [4].

A random variable X is said to have follow Rayleigh distribution with parameter θ if its probability density function (pdf) is given by:

$$g(x) = \theta x e^{-\frac{\theta}{2}x^2} \tag{1}$$

And its corresponding cumulative distribution function (cdf) is given as

$$G(x) = 1 - e^{-\frac{\theta}{2}x^2} \tag{2}$$

where $x > 0, \theta > 0$ where Θ is the scale parameter.

In recent times, a number of authors have developed efficient families of probability distributions and it has been proven that they produce more flexible probability models. These proposed families among others include the quadratic rank transmutation map proposed by [5], the Weibull-X family of distribution by [6], the Weibull-G family of distributions by [7], the Gamma-X family by [8], a Lomax-G family by [9], a new Weibull-G family by [10], a Lindley-G family by [11], a Poisson-X family by [12], a Gompertz-G family by [13], an odd Lindley-G family by [14], extended Poisson family of life distribution by [15] and an odd Lomax generator of distributions by [16].

Adequate utilization of these families and other methods have led to many extensions of the Rayleigh distribution some of which are; the generalized Rayleigh distribution by [17], Bivariate generalized Rayleigh distribution by [18], Transmuted Rayleigh distribution by [19], Weibull-Rayleigh distribution by [20], transmuted Weibull-Rayleigh distribution by [21], the Transmuted Inverse Rayleigh distribution studied by [22] and the odd Lindley-Rayleigh distribution by [23].

Besides the above extensions of the Rayleigh distribution, other models arising from these proposed families of distributions are the odd Lindley inverse exponential distribution by [24], the transmuted normal distribution by [25], the Weibull-Exponential distribution by [26], the transmuted Weibull-exponential distribution by [27], the Weibull-Frechet distribution by [28], the transmuted Lomax distribution by [29], the Gompertz-Lindley distribution by [30], Poisson-exponential distribution by [31], exponential-Poisson distribution by [32] and many others.

Inspired by the above listed families and the related extended probability distributions, this study will propose another extension of the Rayleigh distribution by using the Poisson-X family proposed by [12], this proposed distribution is called "the Poisson-Rayleigh distribution (PRD)".

The rest of this paper is organized in sections as follows: the newly proposed distribution is defined with its plots in section 2. Section 3 presents statistical properties of the new distribution. Section 4 looks at the estimation of parameters using maximum likelihood estimation. An application of the Poisson-Rayleigh distribution and Rayleigh distribution to some real life datasets is presented in section 5 and the final summary and conclusion is provided in section 6.

2 A Poisson-Rayleigh Distribution (PRD)

According to [12], the cdf and pdf of a Poisson-X family of distributions are respectively given by

$$F(x)=\left(1-\mathrm{e}^{-1}\right)^{-1}\left(1-\mathrm{e}^{-[G(x)]^{\alpha}}\right) \tag{3}$$

and

$$f(x)=\alpha\left(1-\mathrm{e}^{-1}\right)^{-1} g(x)\left[G(x)\right]^{\alpha-1} \mathrm{e}^{-[G(x)]^{\alpha}} \tag{4}$$

where $x>0$, and α is the extra shape parameter, $G(x)$ and $g(x)$ are the cdf and pdf of any continuous distribution to be modified respectively.

Putting equation (1) and (2) into equation (3) and (4) and simplifying, we obtain the cdf and pdf of the PRD given in equation (5) and (6) respectively as follows:

$$F(x)=\left(1-\mathrm{e}^{-1}\right)^{-1}\left(1-\mathrm{e}^{-\left[1-\mathrm{e}^{-\frac{\theta}{2}x^{2}}\right]^{\alpha}}\right) \tag{5}$$

and

$$f(x)=\alpha\theta x\left(1-\mathrm{e}^{-1}\right)^{-1} \mathrm{e}^{-\frac{\theta}{2}x^{2}}\left[1-\mathrm{e}^{-\frac{\theta}{2}x^{2}}\right]^{\alpha-1} \mathrm{e}^{-\left[1-\mathrm{e}^{-\frac{\theta}{2}x^{2}}\right]^{\alpha}} \tag{6}$$

where $x>0, \theta>0, \alpha>0,$ α and θ are the parameters of the PRD.

Plots of the pdf and cdf of the PRD using some parameter values are presented in Fig. 1 as follows.

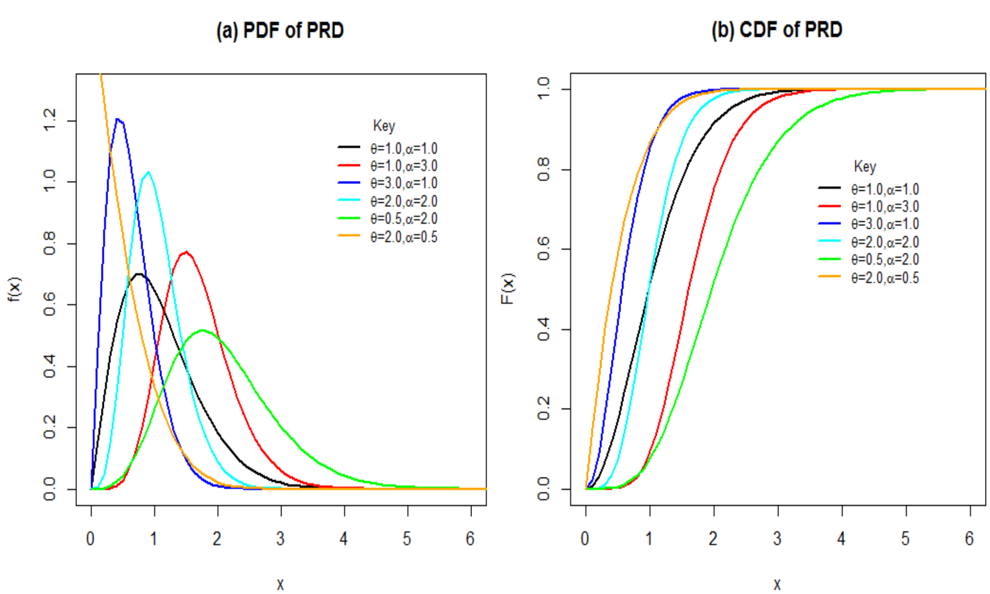

Fig. 1. (a)-PDF and (b)-CDF of the PRD for different values of the parameters

From the figure above, it can be seen that the pdf PRD distribution is positively skewed and takes various shapes depending on the parameter values. Also, from the above plot of the cdf, it is clear that the cdf equals to one when *X* approaches infinity and equals zero when *X* tends to zero as normally expected.

3 Useful Statistical Properties of PRD

In this section, useful properties of the PRD distribution have been derived and discussed as follows:

3.1 Quantile function

According to [33], the quantile function for any distribution is defined in the form $Q(u) = X_q = F^{-1}(u)$ where $Q(u)$ is the quantile function of $F(x)$ for $0 < u < 1$

Taking $F(x)$ to be the cdf of the PRD and inverting it as above will give us the quantile function as follows:

$$F(x) = \left(1-\mathrm{e}^{-1}\right)^{-1}\left(1-\mathrm{e}^{-\left[1-\mathrm{e}^{-\frac{\theta}{2}x^2}\right]^{\alpha}}\right) = u \tag{7}$$

Simplifying equation (7) above and solving for x presents the quantile function of the PRD as:

$$Q(u) = \left\{-\frac{2}{\theta}\ln\left[1-\left(-\ln\left[1-u\left(1-\mathrm{e}^{-1}\right)\right]\right)^{\frac{1}{\alpha}}\right]\right\}^{\frac{1}{2}} \tag{8}$$

This function is used for obtaining some moments like skewness and kurtosis as well as the evaluation of median and for generation of random variables from the distribution.

3.2 Skewness and kurtosis

This paper presents the quantile based measures of skewness and kurtosis due to non-existence of the classical measures in some cases.

According to [34], the Bowley's measure of skewness based on quartiles is given by:

$$SK = \frac{Q\left(3/4\right) - 2Q\left(1/2\right) + Q\left(1/4\right)}{Q\left(3/4\right) - Q\left(1/4\right)} \tag{9}$$

Also, the Moors kurtosis based on octiles proposed by [35] and is given by;

$$KT = \frac{Q\left(7/8\right) - Q\left(5/8\right) - Q\left(3/8\right) + \left(1/8\right)}{Q\left(6/8\right) - Q\left(1/8\right)} \tag{10}$$

where $Q(.)$ is obtainable with the help of equation (8).

3.3 Reliability analysis of the PRD.

The Survival function describes the likelihood that a system or an individual will not fail after a given time. Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \tag{11}$$

Applying the cdf of the PRD in (11), the survival function for the PRD is obtained as:

$$S(x) = 1 - \left(1 - \mathrm{e}^{-1}\right)^{-1} \left(1 - \mathrm{e}^{-\left[1 - \mathrm{e}^{-\frac{\theta}{2}x^2}\right]^{\alpha}}\right) \tag{12}$$

where $x > 0, \alpha, \theta > 0$.

Hazard function is a function that describes the chances that a product or component will breakdown over an interval of time. It is mathematically defined as:

$$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1 - F(x)} \tag{13}$$

Therefore, our definition of the hazard rate of the PRD is given by

$$h(x) = \frac{\alpha\theta x \left(1 - \mathrm{e}^{-1}\right)^{-1} \mathrm{e}^{-\frac{\theta}{2}x^2} \left[1 - \mathrm{e}^{-\frac{\theta}{2}x^2}\right]^{\alpha - 1} \mathrm{e}^{-\left[1 - \mathrm{e}^{-\frac{\theta}{2}x^2}\right]^{\alpha}}}{1 - \left(1 - \mathrm{e}^{-1}\right)^{-1} \left(1 - \mathrm{e}^{-\left[1 - \mathrm{e}^{-\frac{\theta}{2}x^2}\right]^{\alpha}}\right)} \tag{14}$$

where $x > 0, \alpha, \theta > 0$.

The figure below presents a plot of both the survival function (SF) and hazard function (HF) of PRD based on arbitrary parameter values as follows:

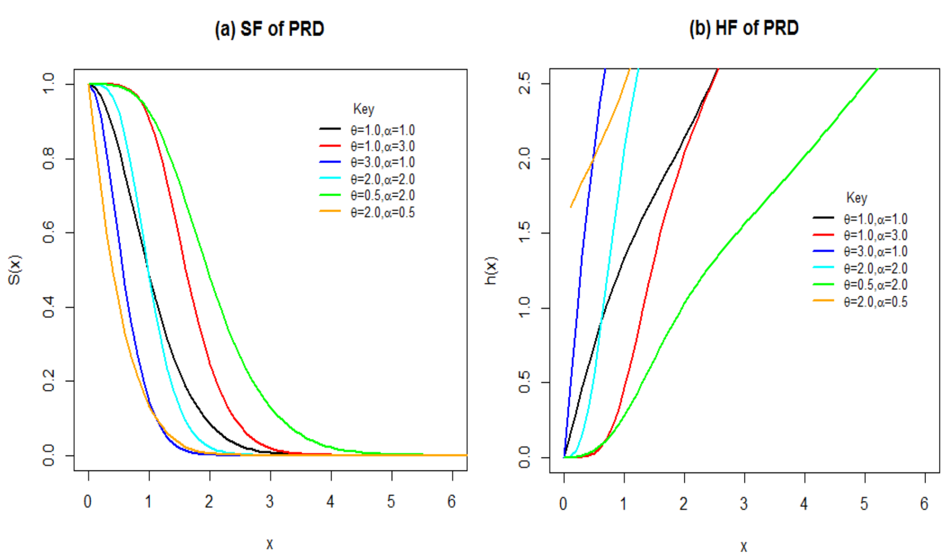

Fig. 2. (a)-SF and (b)-HF of PRD for Selected Values of the Parameters

The plot in Fig. 2(a) show that the chances of survival equal are higher at the beginning or early age and decrease as the time increases and tends to zero at infinity. Fig. 2(b) also revealed that the proposed distribution has increasing failure rate which implies that the probability of failure for any random variable following a PRD increases as time increases, that is, probability of failure or death increases as the process or event progresses.

4 Estimation of Unknown Parameters of the PRD

In this section, the estimation of the parameters of the PRD is done by using the method of maximum likelihood estimation (MLE). Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the PRD with unknown parameters α and θ defined previously.

The likelihood function of the PRD using the pdf in equation (6) is given by;

$$L(x_1, x_2, \ldots, x_n / \alpha, \theta) = (\alpha\theta)^n \left(1-e^{-1}\right)^{-n} e^{-\sum_{i=1}^{n}\left[1-e^{-\frac{\theta}{2}x_i^2}\right]^{\alpha}} \prod_{i=1}^{n}\left\{x_i e^{-\frac{\theta}{2}x_i^2}\left[1-e^{-\frac{\theta}{2}x_i^2}\right]^{\alpha-1}\right\} \tag{15}$$

Let l be the natural logarithm of the likelihood function such that $l = \log L(x_1, x_2, \ldots, x_n \mid \alpha, \theta)$, therefore, taking the natural logarithm of the function above gives:

$$l = n\log\alpha + n\log\theta - n\log\left(1-e^{-1}\right) - \sum_{i=1}^{n}\left[1-e^{-\frac{\theta}{2}x_i^2}\right]^{\alpha} + \sum_{i=1}^{n}\log x_i - \frac{\theta}{2}\sum_{i=1}^{n}\log x_i + (\alpha-1)\sum_{i=1}^{n}\log\left[1-e^{-\frac{\theta}{2}x_i^2}\right] \tag{16}$$

Differentiating l partially with respect to α and θ respectively gives the following results:

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\left[1-e^{-\frac{\theta}{2}x_i^2}\right]^{\alpha}\ln\left[1-e^{-\frac{\theta}{2}x_i^2}\right] + \sum_{i=1}^{n}\log\left[1-e^{-\frac{\theta}{2}x_i^2}\right] \tag{17}$$

$$\frac{\partial l}{\partial\theta} = \frac{n}{\theta} - \frac{\alpha}{2}\sum_{i=1}^{n}x_i^2 e^{-\frac{\theta}{2}x_i^2}\left[1-e^{-\frac{\theta}{2}x_i^2}\right]^{\alpha-1} + \frac{1}{2}\sum_{i=1}^{n}\log x_i + \frac{(\alpha-1)}{2}\sum_{i=1}^{n}\left\{\frac{x_i^2 e^{-\frac{\theta}{2}x_i^2}}{\left[1-e^{-\frac{\theta}{2}x_i^2}\right]}\right\} \tag{18}$$

Equating (17) and (18) to zero (0) and solving for the solution of the non-linear system of equations above will give the maximum likelihood estimates $\hat{\alpha}$ and $\hat{\theta}$ of parameters α and θ respectively. However, these solutions cannot be obtained manually except numerically with the aid of suitable statistical software such as *R* software as used in this study.

5 Applications

In this section, three real life datasets have been considered to check the modeling flexibility of the proposed distribution compared to the Rayleigh distribution. The models to be fitted in this section are the proposed Poisson-Rayleigh distribution (PRD) and the conventional Rayleigh distribution (RD). To identify the most fitted distribution to each of the datasets, the following model selection criteria were used which include the value of the log-likelihood function evaluated at the MLEs (ℓ), Akaike Information Criterion, *AIC,* Consistent Akaike Information Criterion, *CAIC*, Bayesian Information Criterion, *BIC*, Hannan Quin Information Criterion, *HQIC*, Anderson-Darling (A*), Cramèr-Von Mises (W*) and Kolmogorov-smirnov (K-S) statistics. More about these statistics A*, W* and K-S can be seen in [36]. Some of these statistics are computed using the following formulas:

$$AIC = -2\ell + 2k,$$

$$BIC = -2\ell + k\log(n),$$

$$CAIC = -2\ell + \frac{2kn}{(n-k-1)}$$

and

$$HQIC = -2\ell + 2k\log\left[\log(n)\right]$$

Where ℓ denotes the value of log-likelihood function evaluated at the *MLEs*, k is the number of model parameters and n is the sample size. Decisively, the distribution with the lowest values of these criteria is considered to be the most fitted model to the dataset. Also, all the required computations are performed using the R package "AdequacyModel".

Data set I: This is a dataset on the rate of mother-to-child transmission of HIV (Human Immunodeficiency Virus) in Nigeria from the year 2000 to the year 2019. The descriptive statistics and graphical summary of the dataset are also presented.

The mother-to-child HIV transmission rate per 1,000 of population in Nigeria between 2000 and 2019 is as given as follows: 37.35, 37.08, 37.00, 36.98, 36.79, 36.75, 34.35, 32.96, 31.84, 30.35, 30.53, 28.96, 26.71, 22.50, 19.84, 20.04, 19.44, 20.82, 22.09, 22.16

Data source: www.data.unicef.org

The following table presents a summary of the above dataset with some important details:

Table 1. Descriptive statistics for dataset I

n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
20	19.44	22.14	30.44	36.76	29.23	37.35	47.55	-0.18919	-1.55278

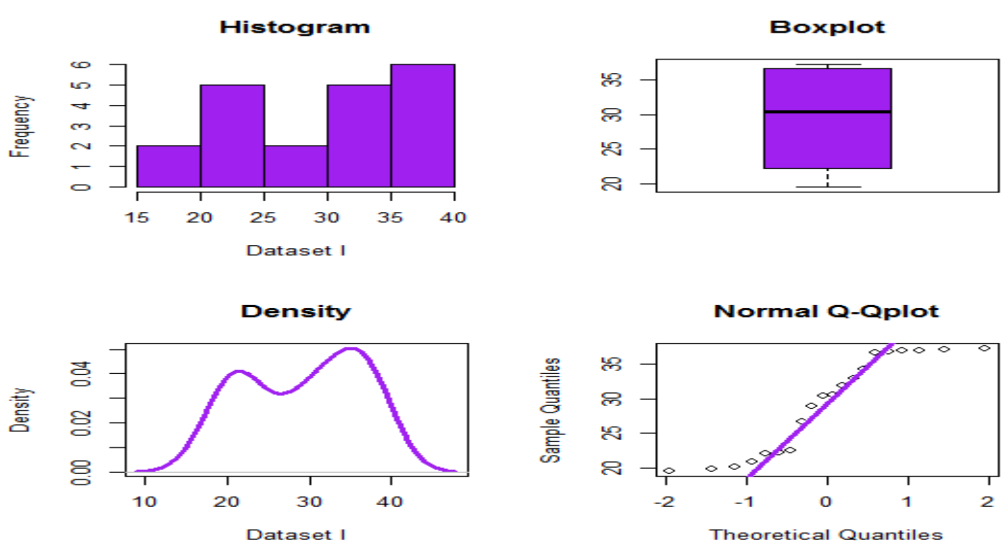

Fig. 3. A graphical summary of dataset I

Following the summary of the descriptive statistics in Table 1 and the histogram, box plot, density and normal Q-Q plot generally referred to as graphical summary in Fig. 3 above, it is seen that the rate of transmission of HIV from mother to child is bimodal and approximately normally distributed.

Applications of the proposed model and the conventional Rayleigh distribution to this data (dataset I) has been done and the results are presented as follows: Table 2 lists the Maximum Likelihood Estimates of the model parameters, Table 3 presents the statistics AIC, CAIC, BIC and HQIC while A*, W* and K-S for the fitted models are given in Table 4.

Table 2. Maximum Likelihood Parameter Estimates for dataset I

Distribution	$\hat{\theta}$	$\hat{\alpha}$
PRD	0.005344705	3.711657673
RD	0.002732482	-

Table 3. The statistics ℓ, AIC, CAIC, BIC and HQIC based on dataset I

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
PRD	71.86771	147.7354	148.4413	149.7269	148.1242	1st
RD	75.69183	153.3837	153.6059	154.3794	153.578	2nd

Table 4. The A^*, W^*, K-S statistic and P-values based on dataset I

Distribution	A^*	W^*	K-S	P-Value (K-S)	Ranks
PRD	1.062499	0.158693	0.3634	0.007127	1st
RD	1.008713	0.1445629	0.40329	0.001888	2nd

The following figure presents a histogram and estimated densities and cdfs of the fitted models to dataset I.

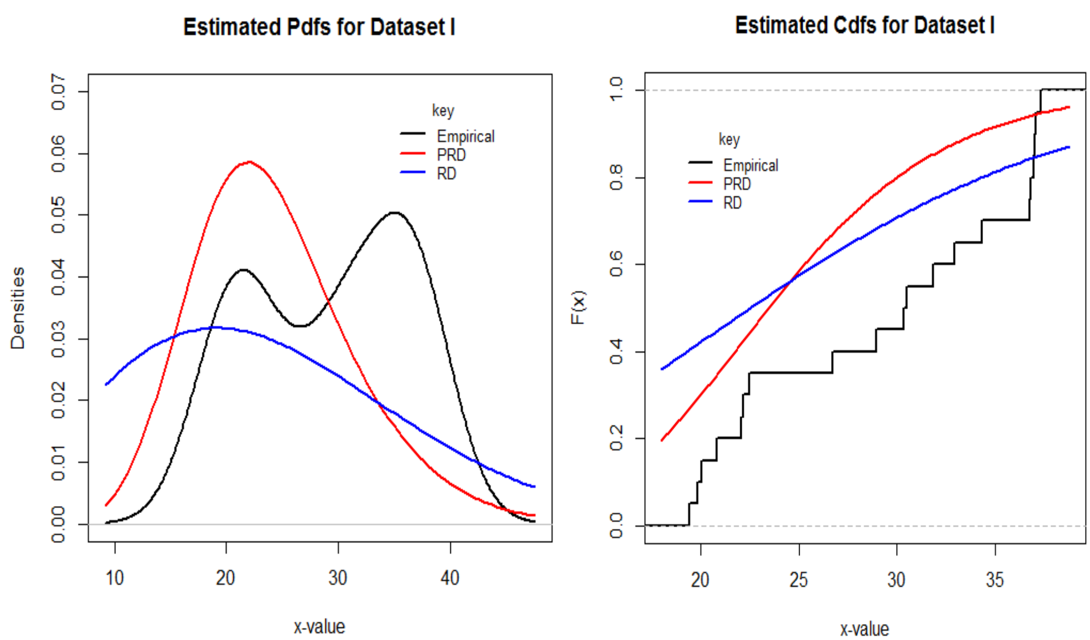

Fig. 4. Estimated densities and cdfs of the fitted distributions to dataset I

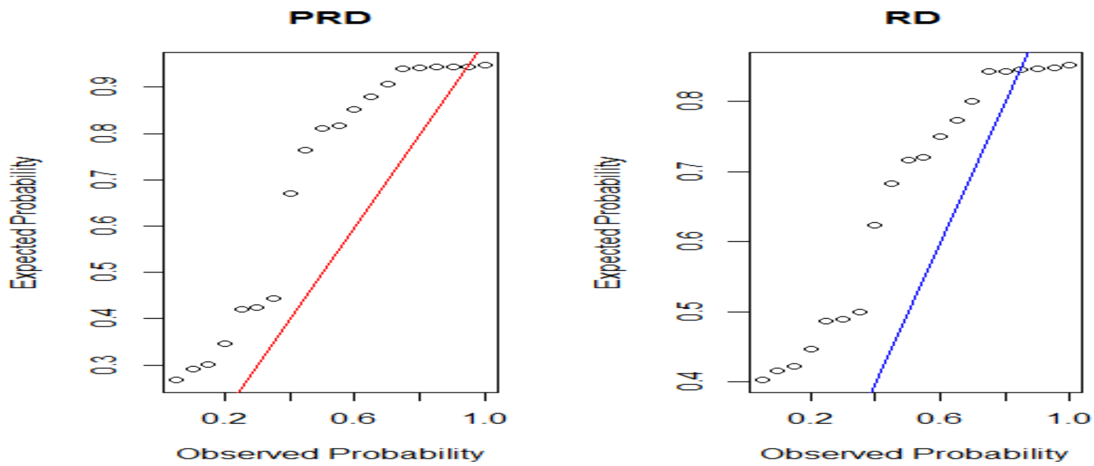

Fig. 5. Probability plots for the fitted distributions based on dataset I

Looking at the results from Table 3, it is revealed that the proposed distribution (PRD) fits dataset I better as compared to the conventional Rayleiyh distribution (RD) using the information criteria (AIC, CAIC, BIC and HQIC). This can also be seen from the statistics in Table 4 which show that the proposed model fits the dataset better than the Rayleigh distribution, this is because the PRD has the minimum values of A^*, W^* and K-S compared to the conventional Rayleigh distribution.

Also, the estimated densities and estimated cumulative distribution functions in Fig. 4 confirm that the proposed model analyses the dataset better than the conventional RD. Similarly, the probability plots presented in Fig. 5 shows that the proposed distribution (PRD) is more flexible than the RD as already revealed previously in Table 3 and 5.4 as well as Fig. 4.

Data set II: This is a real life dataset and it represents the strength of 1.5cm glass fibers initially collected by members of staff at the UK national laboratory. It has been used by [37,38,7,39,27] as well as [40]. Its summary is given as follows:

Table 5. Descriptive statistics of dataset II

n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
63	0.550	1.375	1.590	1.685	1.507	2.240	0.105	-0.8786	3.9238

Considering the descriptive statistics in Table 5 and the graphical summary in Fig. 6 above, it is observed that the real life data (dataset II) is negatively skewed, that is, skewed to the left.

Again the applications of the proposed model and the conventional Rayleigh distribution to this data (dataset II) has been done and the results are presented as follows: Table 6 lists the Maximum Likelihood Estimates of the model parameters, Table 7 presents the statistics AIC, CAIC, BIC and HQIC while A*, W* and K-S for the fitted models are given in Table 8.

Table 6. Maximum Likelihood Parameter Estimates for dataset II

Distribution	$\hat{\theta}$	$\hat{\alpha}$
PRD	1.545945	4.891722
RD	0.8423323	-

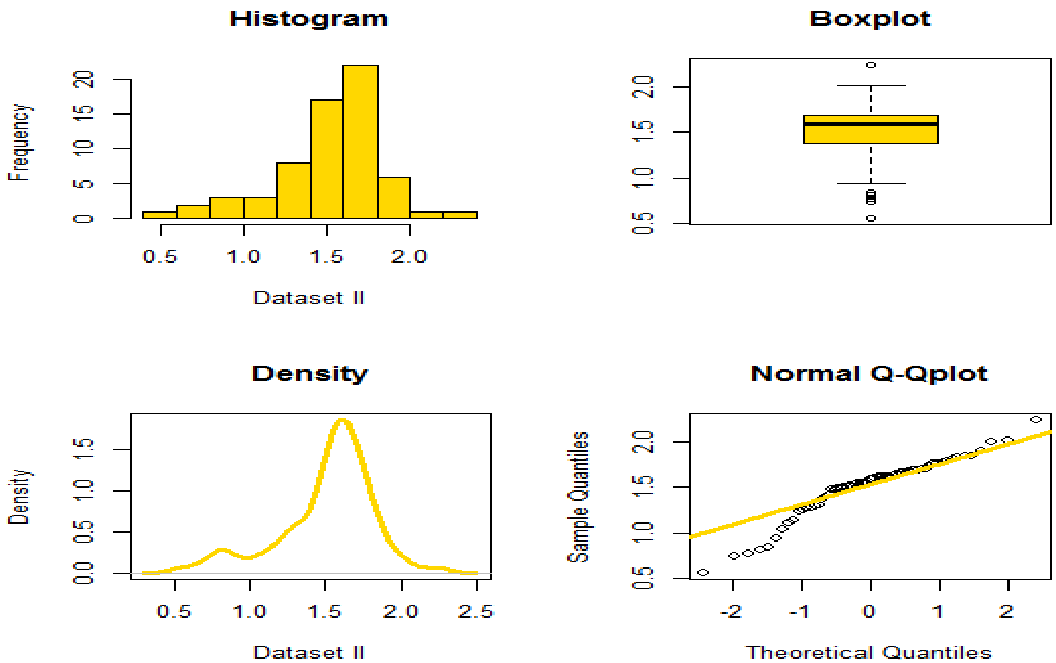

Fig. 6. A graphical summary of dataset II

Table 7. The statistics $\hat{\ell}$, AIC, CAIC, BIC and HQIC based on dataset II

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
PRD	25.5539	55.10779	55.30451	59.42556	56.80878	1st
RD	50.53607	103.0721	103.1367	105.231	103.9226	2nd

Table 8. The A^*, W^*, K-S statistic and P-values based on dataset II

Distribution	A^*	W^*	K-S	P-Value (K-S)	Ranks
PRD	3.389985	0.6187591	0.20796	0.007888	1st
RD	2.646397	0.483948	0.33607	1.054e-06	2nd

The following figure presents the estimated densities and cdfs of the fitted models to dataset II.

Considering the results from Table 7, it is seen that the new distribution (PRD) fits dataset II better than the conventional Rayleigh distribution (RD) using the information criteria (AIC, CAIC, BIC and HQIC). The same result is found from the statistics in Table 8 indicating that the proposed model (Poisson-Rayleigh distribution, PRD) fits dataset II better than the conventional Rayleigh distribution, this is as a result of the fact that the PRD has the lowest values of A^*, W^* and K-S compared to the conventional Rayleigh distribution.

More so, the estimated densities and estimated cumulative distribution functions in Fig. 7 prove that the proposed model describes the second dataset better than the conventional RD. Conclusively, the probability plots displayed in Fig. 8 show that the proposed distribution (PRD) is more flexible compared to the RD as already demonstrated in Table 7 and 8 as well as Fig. 6.

Data set III: This dataset is on the gauge length of 10mm obtained from [41]. The data set holds sixty-three observations as: 1.901, 2.132, 2.203, 2.228, 2.257, 2.350, 2.361, 2.396, 2.397, 2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614, 2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.740, 2.856, 2.917, 2.928, 2.937, 2.937, 2.977, 2.996, 3.030, 3.125, 3.139, 3.145, 3.220, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346, 3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628, 3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, 5.020. The following table presents a summary of the above dataset:

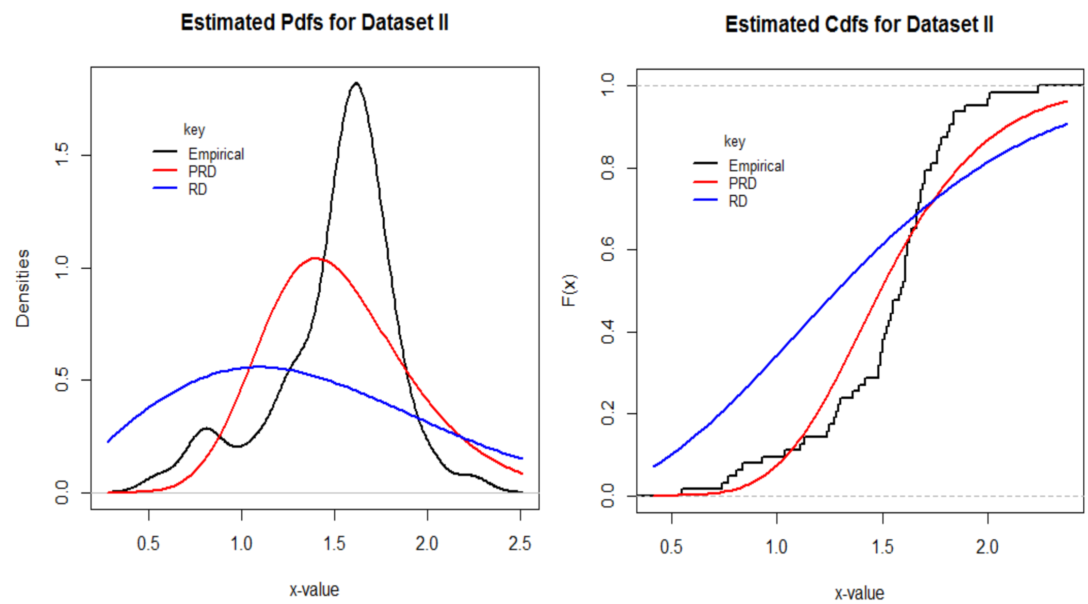

Fig. 7. Estimated densities and cdfs of the fitted distributions to dataset II

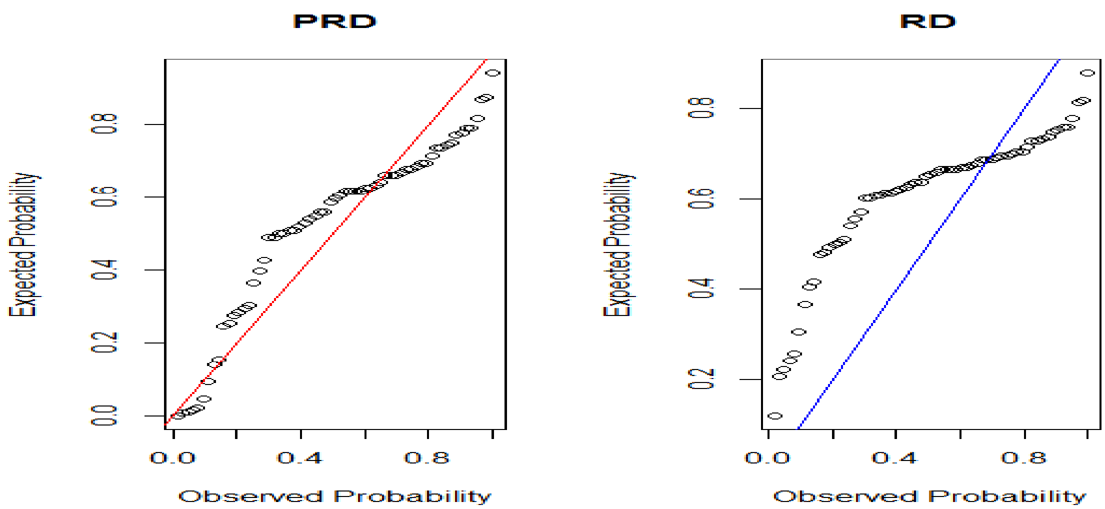

Fig. 8. Probability plots for the fitted distributions based on dataset II

Table 9. Descriptive Statistics for dataset III

n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
63	1.901	2.554	2.996	3.421	3.059	5.020	0.38554	0.63285	0.28634

Based on the descriptive statistics in Table 9 and the histogram, box plot, density and normal Q-Q plot or the graphical summary in Fig. 9 above, it is seen that the third dataset is skewed to the right or it is positively skewed.

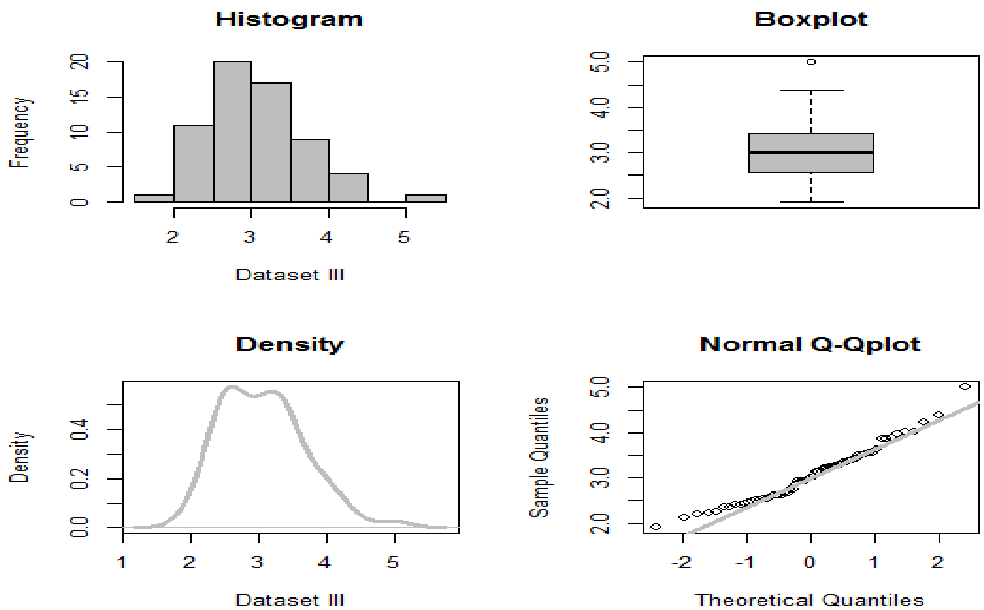

Fig. 9. A graphical summary of dataset III

Another application of the new distribution and the conventional Rayleigh distribution to this third data (dataset III) has been done and the results are shown in tables as follows: Table 10 lists the Maximum Likelihood Estimates of the model parameters, Table 11 presents the statistics AIC, CAIC, BIC and HQIC while A*, W* and K-S for the fitted models are given in Table 12.

Table 10. Maximum likelihood parameter estimates for dataset III

Distribution	$\hat{\theta}$	$\hat{\alpha}$
PRD	0.3455407	4.6768841
RD	0.2050128	-

Table 11. The statistics ℓ, AIC, CAIC, BIC and HQIC based on dataset III

Distribution	$\hat{\ell}$	**AIC**	**CAIC**	**BIC**	**HQIC**	**Ranks**
PRD	62.67903	129.3581	129.5581	133.6443	131.0439	1st
RD	93.52005	189.0401	189.1057	191.1832	189.883	2nd

Table 12.The A^*, W^*, K-S statistic and P-values based on dataset III

Distribution	A^*	W^*	**K-S**	**P-Value (K-S)**	**Ranks**
PRD	0.3589833	0.06089162	0.14973	0.1186	1st
RD	0.4380517	0.0663365	0.3602	1.59e-07	2nd

The following figure presents the estimated densities and cdfs of the fitted models to dataset III.

Based on the results from Table 11, one can see that the Poisson-Rayleigh distribution (PRD) fits dataset III much better than the conventional Rayleigh distribution (RD) following these information criteria (AIC, CAIC, BIC and HQIC). This same performance is also discovered with the statistics in Table 12 which is an indication that the proposed model fits the third dataset (dataset III) better than the conventional Rayleigh distribution, since it has the minimum values of A^*, W^* and K-S compared to the other fitted model.

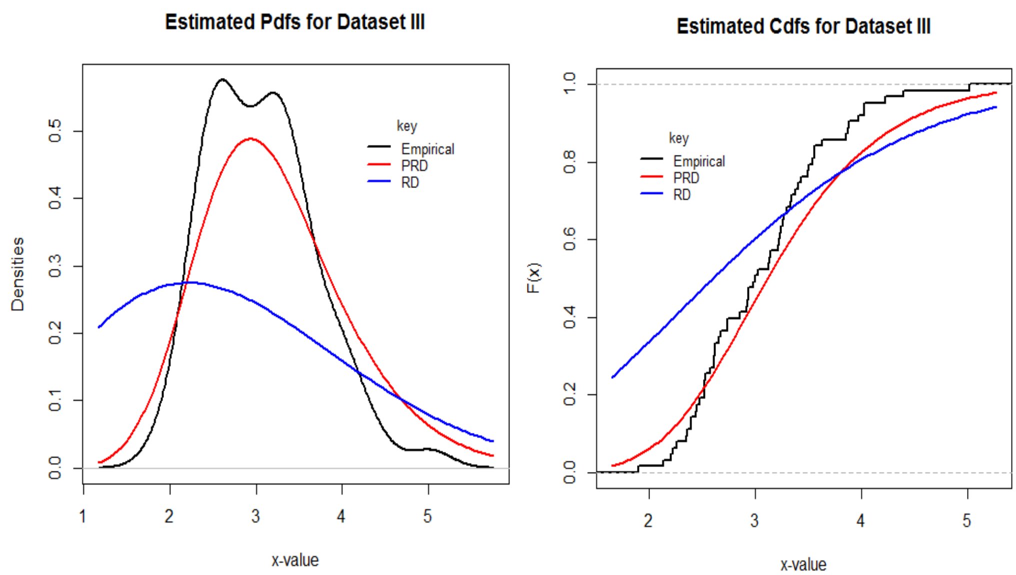

Fig. 10. Estimated densities and cdfs of the fitted distributions to dataset III

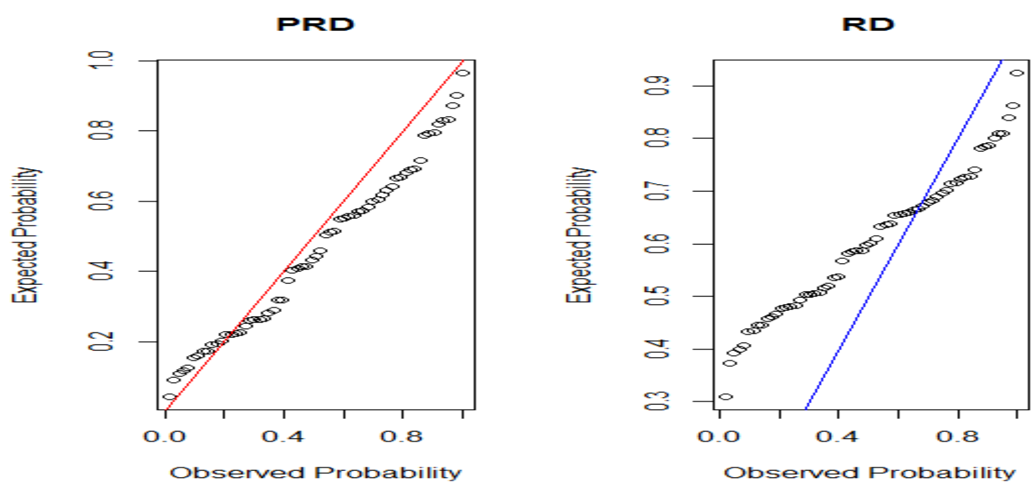

Fig. 11. Probability plots for the fitted distributions based on dataset III

Also, the fitted densities and cumulative distribution functions in Fig. 10 also confirm that the proposed model fits the dataset (dataset III) better than the conventional RD. Again, the probability plots in Fig. 11 show that the proposed distribution (PRD) is more flexible than the other fitted distribution (RD) as already seen in Table 11 and 5.12 as well as in Fig. 10.

Finally, this whole analysis has proven the general statement that adding parameter(s) to any continuous probability distribution always lead to a distribution with greater flexibility in modeling real life data as reported by many other authors in previous studies. This also shows that the Poisson-X family by [12] is good for developing new continuous distributions.

6 Conclusion

This paper developed a new distribution called “a Poisson-Rayleigh distribution”. The statistical properties of this model which are useful have been derived and studied. The quantile function, coefficient of skewness and kurtosis, survival function and hazard function were defined and discussed in this paper. The unknown parameters of the proposed model were estimated using the method of maximum likelihood estimation. The PRD was used to fit real life datasets. Results from the application of the proposed model to the real life datasets reveal that the Poisson-Rayleigh distribution fits the datasets much better than the Rayleigh distribution. This performance of our model is an indication that the proposed model will be useful for describing other real life situations.

Competing Interests

Authors have declared that no competing interests exist.

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